DIFFUSION OF INNOVATIONS

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Diffusion of Innovations

- Innovations (even the beneficial ones)
  - contact with innovation is not sufficient to spread it
  - may not diffuse through the society- or may do it very slowly.
  - simple epidemic models (spread through contact) do not apply
- Understanding diffusion of innovations is crucial and has several applications:
  - diffusion of new products, technologies,
  - diffusion of new policies,
  - diffusion of ideas.

Linear Threshold Model

- Proposed by Granovetter in *Threshold models of collective behavior* [1978],
- Innovation emerges in a group of nodes (the seed set).
- Each individual has a certain threshold.
- An individual adopts the innovation if the fraction of her neighbors that have adopted is above a certain threshold.
- Granovetter argues the model explains several aggregate level behaviors such as
  - diffusion of innovations,
  - spread of rumors and diseases,
  - riot behaviors,
  - migration.

Linear Threshold Model

- Valente [1996] empirically tests the model:
  - diffusion of a prescription drug among medical doctors.
  - the actual behavior could be captured by the model.
  - the model explains diffusion of innovations with relatively low sunk cost (fads, fashion, etc.)
- Kempe [2003, 2005] studies the model under random thresholds
  - Determining the *optimal* seed set is NP-hard.
- Watts [2002] studies the model under random thresholds and on random graphs
  - Discusses significant mass of early adopters is crucial for successful diffusion.
- No significant analysis - intuition has been proposed regarding diffusion behavior, thresholds, and the network structure so far.

The Network Model

- Consider a graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$,
  - $\mathcal{V}$ is the set of individuals, and $\mathcal{E}$ is the set of links among individuals.
- Define neighbor set of individual i as:

$$\mathcal{N}_i(\mathcal{G}) = \{j | (j, i) \in \mathcal{E}\} .$$

- $\mathcal{N}_i$ represent set of individuals who can influence i.
- Each individual i has a threshold $\phi_i \in [0, 1]$.
- At time $k = 0$, a subset of individuals are selected as the seed set $\Phi(0) \subset \mathcal{V}$.
  - group of innovators who have already been exposed to innovation,
  - the set of promoters who have certain social, economic and/or political agenda.

The Threshold Model

- At the next iteration, an individual $i \notin \Phi(0)$ will adopt the innovation if
  - at least ϕ_i fraction of her neighbors are in the seed set, *i.e.*,

$$\frac{|\Phi(0) \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} \geq \phi_i \Rightarrow i \in \Phi(1).$$

- The set of new adopters is denoted by $\Phi(l)$
- In general, an individual $i \notin \bigcup_{l=0}^{k-1} \Phi(l)$ will adopt the innovation at iteration k if

$$\frac{|\bigcup_{l=0}^{k-1} \Phi(l) \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} \geq \phi_i \Rightarrow i \in \Phi(k).$$

Questions

- Will the innovation go "viral"?
- If not, what are the fixed points?
- Which one of the fixed points will be selected give an initial seed set ?

Coherent Sets

Definition

A nonempty subset $\mathcal{M} \subset \mathcal{V}$ is called a coherent set if;

$$\frac{|\mathcal{M} \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} > 1 - \phi_i \text{ for all } i \in \mathcal{M}. \tag{1}$$

- For each member of the set, the fraction of neighbors residing in the set is above an agent specific threshold!
- Coherence measures how well members of a given set are connected among themselves.
- Members of a coherent set can not adopt the innovation unless there exists at least one adopter inside the set itself.

An Example

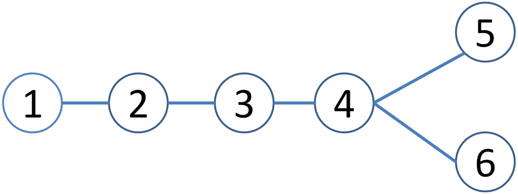

Figure: A sample network.

- $\phi_i = 0.5 - \epsilon$, $i = 1, 2$, and $\phi_i = 0.5 + \epsilon$, $i = 3, 4, 5, 6$, where $0 < \epsilon << 1$.
- Multiple coherent sets including:
  - $\{1, 2, 3\}, \{3, 4, 5, 6\}, \{4, 5, 6\}, \{1, 2, 3, 4, 5\}, \{1, 2, 3, 4, 6\}$.
- Coherent set is not unique for a given configuration!

Fixed points

Lemma

For a given graph $\mathcal{G}$ and threshold values $\{\phi_i\}_{i\in\mathcal{V}}$ the adopter set $\Phi^\star$ is a fixed point if and only if its complement $(\Phi^\star)^c$ is a coherent set.

Proof:

$$\frac{|(\Phi^\star)^c \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} = \frac{|\mathcal{N}_i(\mathcal{G})| - |\Phi^\star \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} > 1 - \phi_i$$

which implies:

$$\frac{|\Phi^\star \cap \mathcal{N}_i(\mathcal{G})|}{|\mathcal{N}_i(\mathcal{G})|} < \phi_i$$

Final Adopter Set

Proposition

For a given graph $\mathcal{G}$, threshold values $\{\phi_i\}_{i\in\mathcal{V}}$ and seed set $\Phi(0)$, one of the following holds:

1. *If there exists no $\Phi^\star \subset \mathcal{V}$ such that $\Phi(0) \subseteq \Phi^\star$ and $(\Phi^\star)^c$ is coherent, then the innovation diffuses through the whole society.*
2. *If there exists a unique $\Phi^\star$ such that $\Phi(0) \subseteq \Phi^\star$ and $(\Phi^\star)^c$ is coherent, then only the set $\Phi^\star$ adopts the innovation.*
3. *If there exists multiple subsets, i.e., $K > 1$ and $\{\Phi^\star_s\}_{s=1}^K$, such that $\Phi(0) \subseteq \Phi^\star_s$ for all s and $(\Phi^\star_s)^c$ for all s are coherent, then the set:*

$$\Phi^\star = \bigcap_{s=1}^{K} \Phi^\star_s,$$

adopts the innovation.

An Example

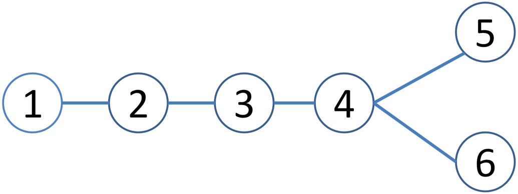

Figure: A sample network.

- $\phi_i = 0.5 - \epsilon$, $i = 1, 2$, and $\phi_i = 0.5 + \epsilon$, $i = 3, 4, 5, 6$, where $0 < \epsilon << 1, \Phi(0) = \{1\}$.
- Final adopter set $\{1, 2\}$.
- It is uniquely defined.

Discussions

- Existing intuition:
  - A network which consists of large number of small clusters might be more advantageous for innovation diffusion.
  - Locally dense clusters might reinforce adoption through short cycles.
- Discussed through experiments and simulations (Centola[2010])
- Our results suggest that:
  - Clustering may weaken adoption due to coherence.
    - ★ Each small cluster might form a coherent set by itself!
- The effect of clusters is not trivial.
- An effective seed set should be chosen such that:
  - The remaining part of the society does not have large coherent sets.

Conclusion

- Studied linear threshold model with individual specific thresholds.
- Characterized the final adopter set in terms of coherent sets.
- Final adopter set is a function of the seed set, network structure and the thresholds.
- Effect of the network structure is not trivial.
- Clustering has both reinforcing and weakening effects on the diffusion.