Performance bounds for tracking in a multipath environment

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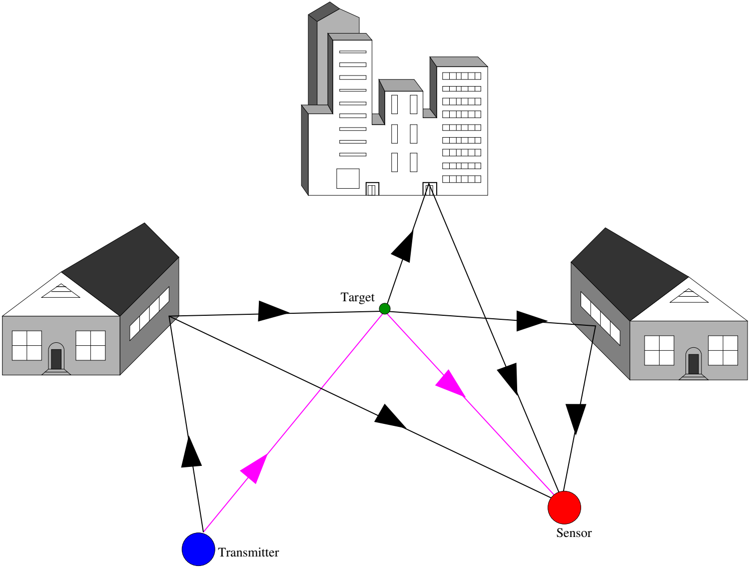
Outline

1. Introduction
2. Existing work
3. Assumptions and extensions
4. Modelling and notation
5. Performance bounds
6. Future Work
7. Q&A

Tracking a target in a multipath environment



Tracking a target in a multipath environment Contd...

- Conventional radar systems
  - Rely on **Line of Sight** (LOS) communication between the target and the radar head
  - Treat multipath as interference
- Radar operating in an urban environment
  - LOS is no longer guaranteed.
  - Lot of reflections from obstacles
- Modern research interests on multipath

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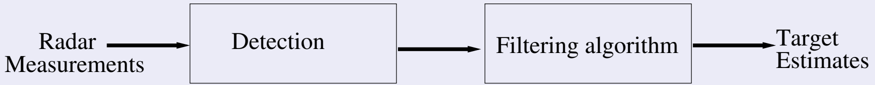
Existing work

- Common assumptions (Krolik *et al* 2006, Chakraborty *et al* 2010,)
  - Knowledge of the wall locations
  - Knowledge of the wall reflectivities
  - Knowledge of the number of targets
  - Too specific geometric assumptions regarding the obstacles (eg. narrow canals)
- Detection based tracking (Barbosa *et al* 2008)



Assumptions and extensions

We derive the lower bound on the MSE under the following:

- Common assumptions
  - Target is a point scatterer
  - Building locations are known
  - Specular reflections at walls (reflective angle=incident angle)
  - Higher order reflections are ignored
  - Multiple transmitters and receivers
  - Each receiver consists of a phase array antenna with L number of elements
- Extensions
  - Reflectivity of Buildings modelled as random variables
  - Each multipath is subject to a random phase shift which is distributed according to a uniform distribution
  - Tracking performed with pre-detection measurements
  - No geometrical restrictions on the wall placement

Modelling and notation

- The state space at time t_k could be partitioned into 3 components
  - Target dynamics; $\boldsymbol{x}_k = [x_k\ \dot{x}_k\ y_k\ \dot{y}_k]'$
  - Wall and Target reflectivities; $\boldsymbol{z} = [\epsilon\ \varepsilon_1 \cdots \varepsilon_B]'$
  - Collection of random phases that affect each path; ψ_k
- Target reflectivity(ϵ) and building reflectivities ($\varepsilon_1 \cdots \varepsilon_B$) are modeled as Gaussian random parameters

- State evolves according to;

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{G}_k \mathbf{w}_k, \qquad k = 1, 2, \ldots,$$
$$\mathbf{z} \sim N(\mu_z, P_z)$$
$$\psi_k \sim U_{(0:2\pi)^{P(\mathbf{x}_k)}}$$

Where $P(\mathbf{x}_k)$ denotes the number of multipaths at time k

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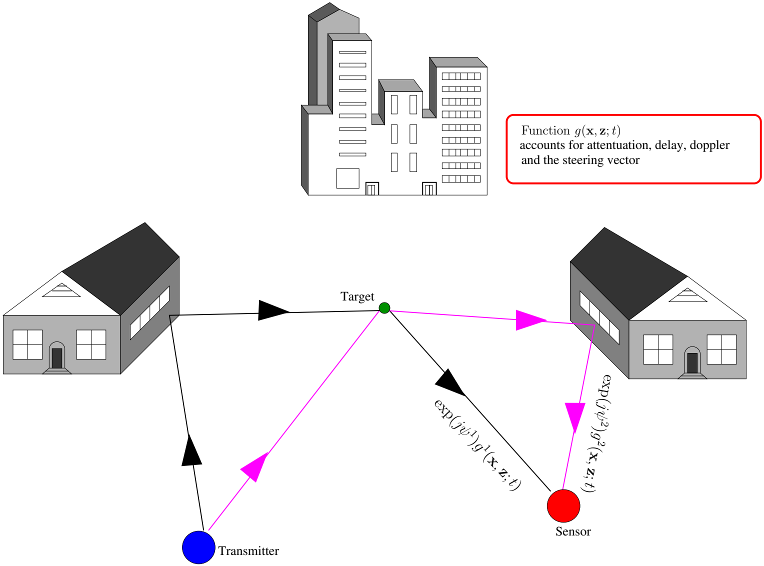
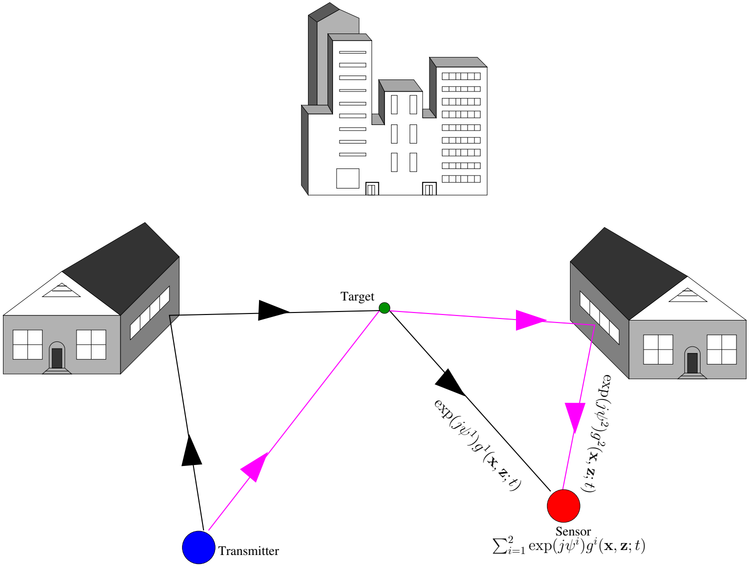
Measurement model



Measurement model Contd...



Posterior Cramer-rao lower bound (PCRB)

- Let **x** be a vector of random parameters and **y** be a vector of measured data.
- Let $g(\mathbf{y})$ be an estimate of **x**. The PCRB has the form

$$E[g(\mathbf{y}) - \mathbf{x}][g(\mathbf{y}) - \mathbf{x}]' \geq J^{-1}$$

Where J is the Information matrix

- Recursive method proposed by Tichavsky *et al* (1998) to obtain the PCRB
- Challenging to find the various quantities needed to apply this method for the setup we have used
- How this was done is discussed next

Performance bounds

Let $\bar{\mathbf{X}}_k = [\mathbf{X}_k' \;\; \psi_k']'$ The lower bound for MSE of estimators of $\bar{\mathbf{X}}_k$ and $\mathbf{Z}_k$, denoted as $(\mathbf{J}_k^{\bar{x}\bar{x}})^{-1}$ and $(\mathbf{J}_k^{zz})^{-1}$ could be found as;

$$\mathbf{J}_k^{\bar{x}\bar{x}} = \mathbf{H}_k^{33} - (\mathbf{H}_k^{13})'[\mathbf{J}_{k-1}^{xx} + \mathbf{H}_k^{11}]^{-1}\mathbf{H}_k^{13}$$
$$\mathbf{J}_k^{\bar{x}z} = (\mathbf{H}_k^{23})' - (\mathbf{H}_k^{13})'[\mathbf{J}_{k-1}^{xx} + \mathbf{H}_k^{11}]^{-1}\mathbf{J}_{k-1}^{xz}$$
$$\mathbf{J}_k^{zz} = \mathbf{J}_{k-1}^{zz} + \mathbf{H}_k^{22} - (\mathbf{J}_{k-1}^{xz})'[\mathbf{J}_{k-1}^{xx} + \mathbf{H}_k^{11}]^{-1}\mathbf{J}_{k-1}^{xz}$$

where, with $\mathbf{Q}_k = \mathbf{G}_k\mathbf{G}_k'$,

$$\mathbf{H}_k^{11} = \text{diag}(\mathbf{F}_k'\mathbf{Q}_k^{-1}\mathbf{F}_k, [\mathbf{0}]_{P(\mathbf{x}_k)})$$
$$\mathbf{H}_k^{13} = -\text{diag}(\mathbf{F}_k'\mathbf{Q}_k^{-1}, [\mathbf{0}]_{P(\mathbf{x}_k)})$$
$$\mathbf{H}_k^{22} = \frac{1}{\sigma^2}\text{Re}\{\text{E}[\nabla_{\mathbf{z}_k} h(\cdot)'(\nabla_{\mathbf{z}_k} h(\cdot)^*)']\}$$
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with ∇ the gradient operator and $p(\mathbf{x_k})$ being the number of paths.

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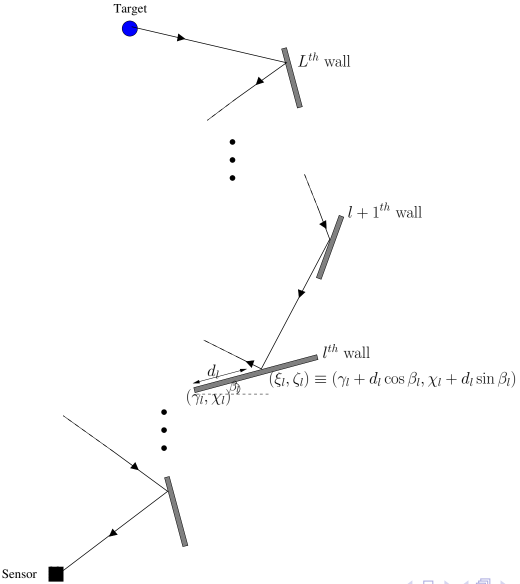
Finding derivatives



Finding derivatives Contd...

- We need to evaluate partial derivatives with respect to **x** of functions of the form:

$$f(d_1(\mathbf{x}), \ldots, d_L(\mathbf{x}), \delta_1(\mathbf{x}), \ldots, \delta_S(\mathbf{x}), \mathbf{z})$$

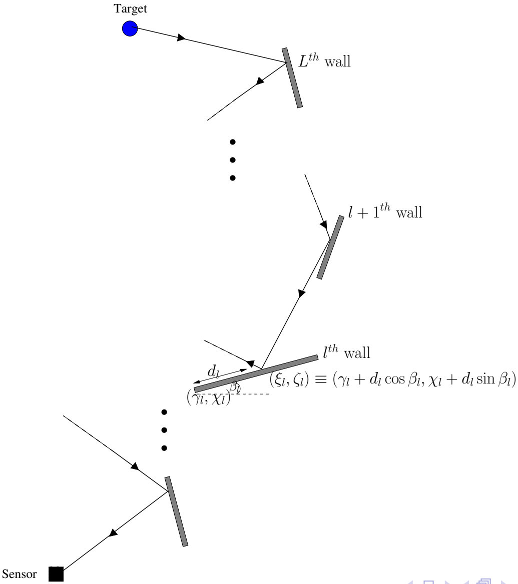
Where δ_s is the distance from the s^{th}reference and reflection points in the transmitter-target path

- Derivative of f with respect to x could be calculated as;

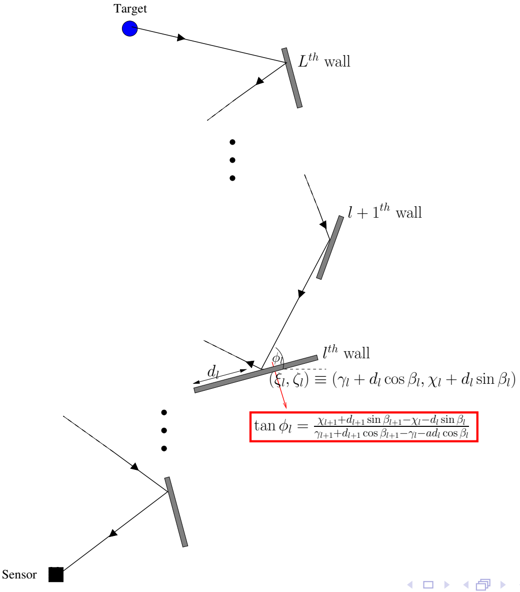
$$\frac{\partial f}{\partial x} = \sum_{i=0}^{L} \frac{\partial f}{\partial d_i} \frac{\partial d_i}{\partial x} + \sum_{j=0}^{S} \frac{\partial f}{\partial \delta_j} \frac{\partial \delta_j}{\partial x}$$

- How to calculate $\frac{\partial d_i}{\partial x}$ and $\frac{\partial \delta_j}{\partial x}$???

Finding derivatives Contd...

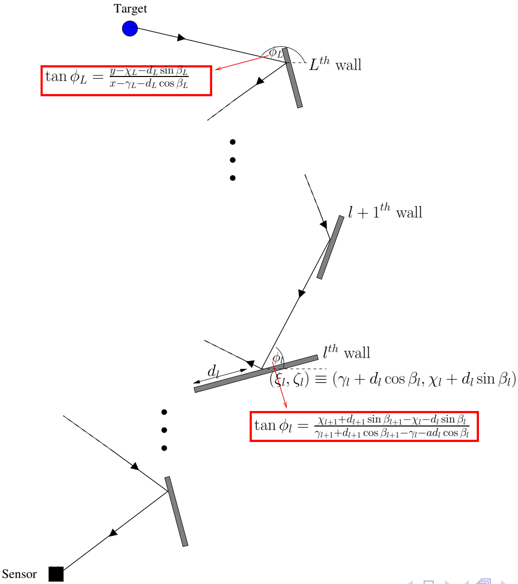


Finding derivatives Contd...



$$\tan\phi_l = \frac{\chi_{l+1}+d_{l+1}\sin\beta_{l+1}-\chi_l-d_l\sin\beta_l}{\gamma_{l+1}+d_{l+1}\cos\beta_{l+1}-\gamma_l-ad_l\cos\beta_l}$$

Finding derivatives Contd...



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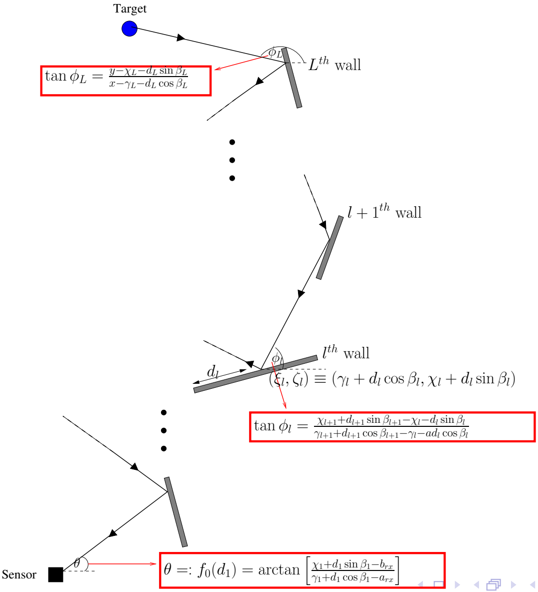


Finding derivatives Contd...

- Three equations obtained

$$
\begin{aligned}
d_L &= f_L(x, y, \theta) \\
d_l &= f_l(d_{l+1}, \theta) \qquad \text{for } l = 1, \ldots, L-1 \\
\theta &= f_0(d_1)
\end{aligned}
$$

- Recursive relationship for the partial derivatives could be expressed as; $\frac{\partial d_l}{\partial x}$ for $l = 1, \ldots, L$ as;

$$\frac{\partial d_l}{\partial x} = \alpha_l + \eta_l \frac{\partial \theta}{\partial x}$$

with $\alpha_l = \alpha_{l+1} \partial f_l / \partial d_{l+1}$ and $\eta_l = \eta_{l+1} \partial f_l / \partial d_{l+1} + \partial f_l / \partial \theta$ where $\alpha_L = \partial f_L / \partial x$ and $\eta_L = \partial f_L / \partial \theta$

- Once α_1 and η_1 is known $\frac{\partial \theta}{\partial x}$ could be obtained

Illustration

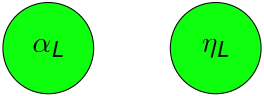
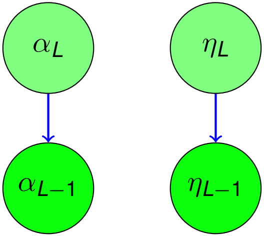


Illustration Contd...



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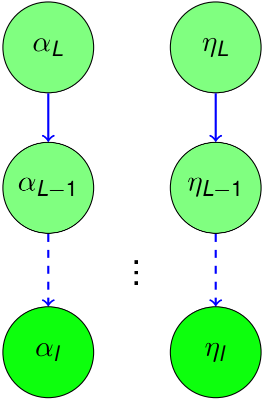

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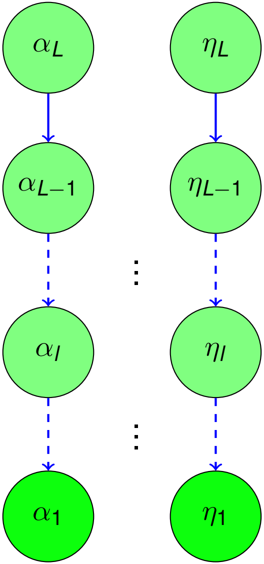


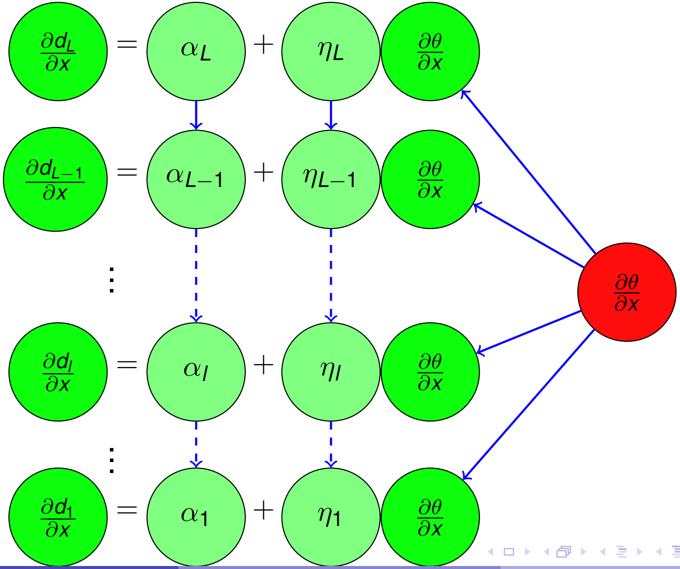
Illustration Contd...

- Now we can find α_0 and η_0 and thus $\frac{\partial\theta}{\partial x}$

$$\alpha_0 = \alpha_1 \frac{\partial f_0}{\partial d_1} \text{ and } \eta_0 = \eta_1 \frac{\partial f_0}{\partial d_1}$$

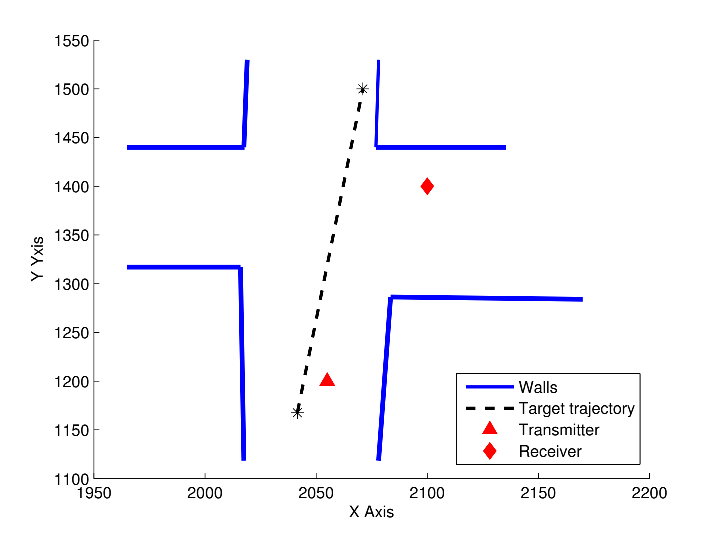
$$\frac{\partial\theta}{\partial x} = \frac{\alpha_0}{1 - \eta_0}$$

Illustration Contd...



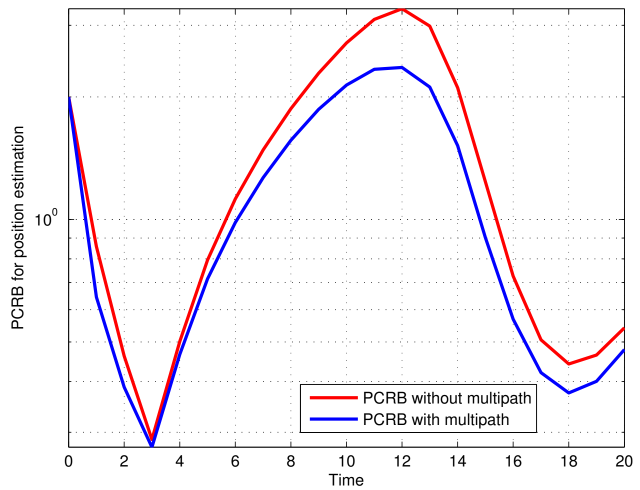
Performance bounds Contd...

Radar scene



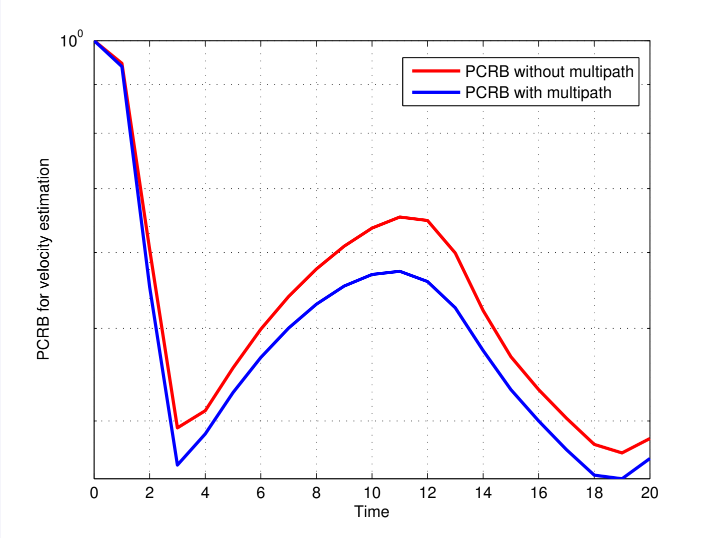
Performance bounds Contd...

Results (PCRB of positions estimates)



Performance bounds Contd...

Results (PCRB of velocity estimates)



Future work

- Implement a filter for the setup introduced
- Multiple targets and possibly unknown number of targets
- Unknown locations of the obstacles
- Waveforms that would give better results
- Resource scheduling

Q&A

Thank you!