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# A Kalman-like Algorithm with no Requirements for Noise and Initial Conditions


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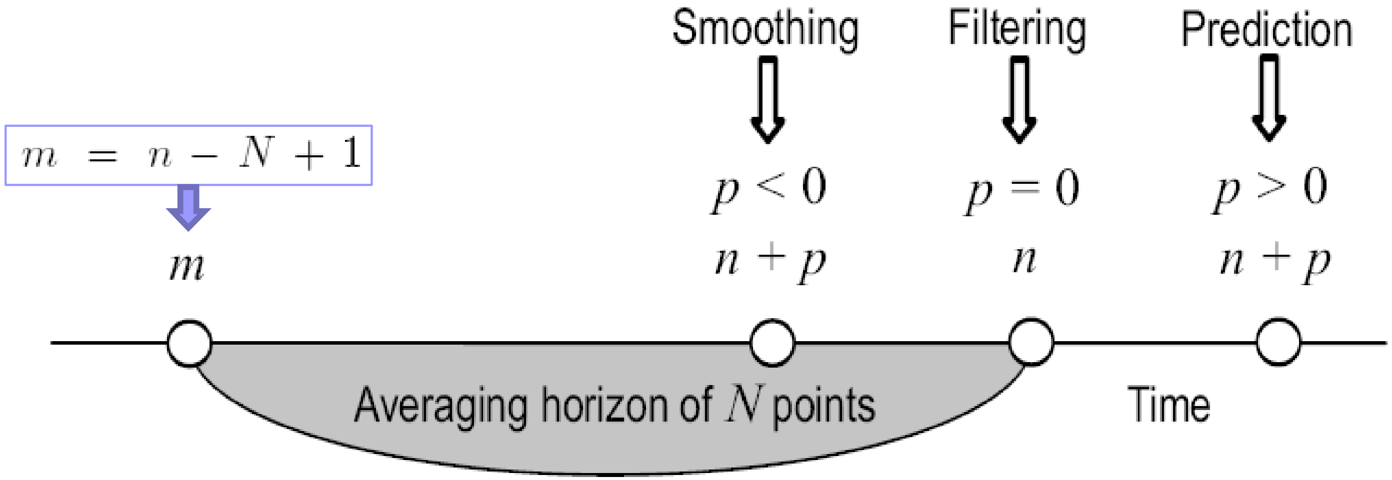
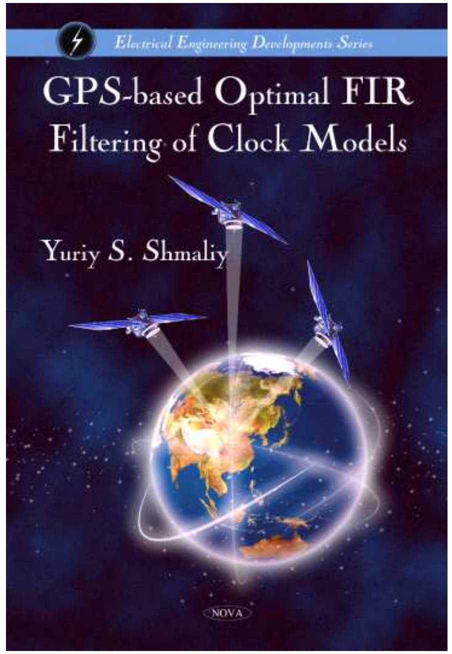
# Outline

# 1. FIR Estimation

The following *estimation problems* can be solved with *FIR structures* on an averaging horizon of *N* past points



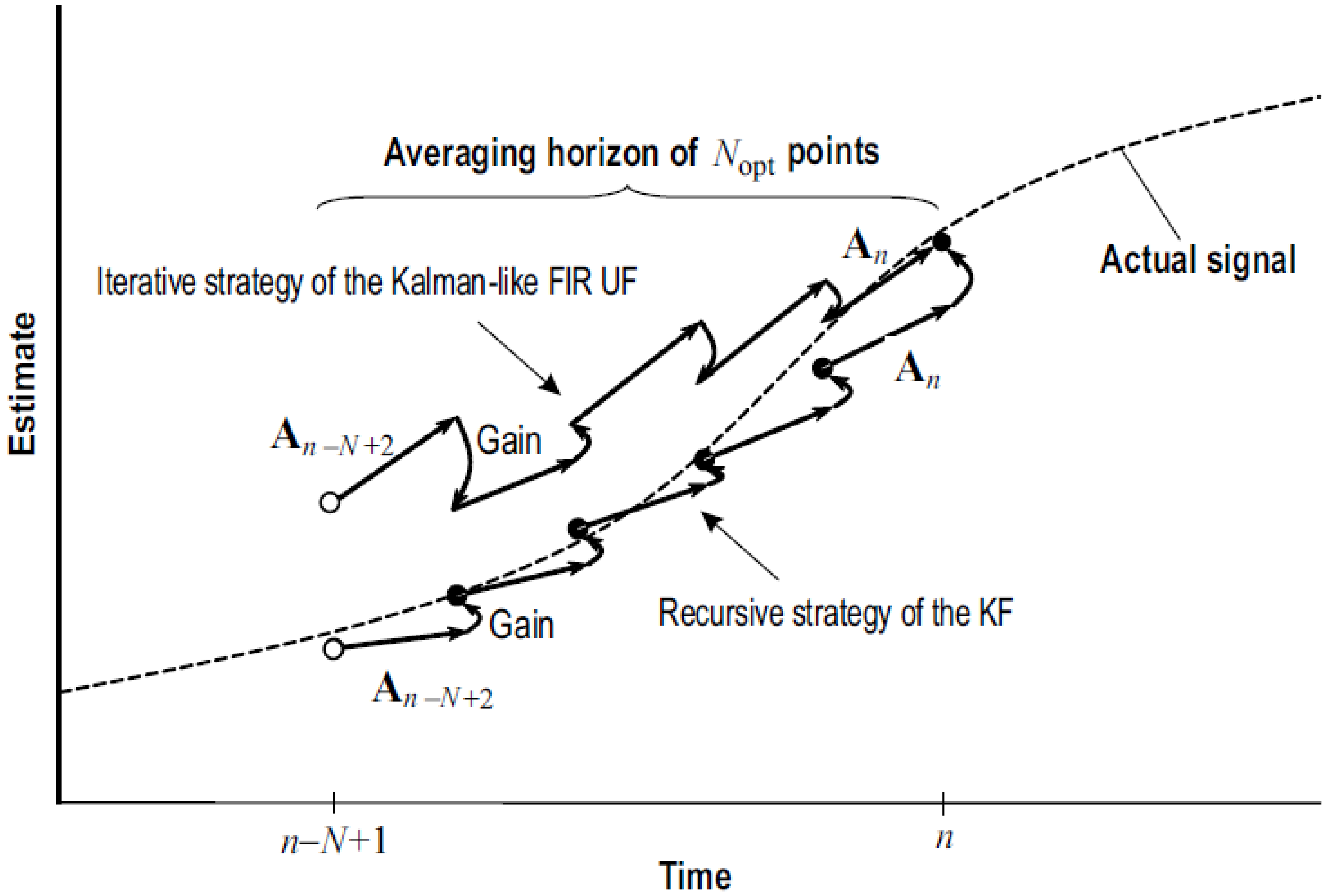

The *FIR estimator* can be designed in either the *batch* or *iterative Kalman-like* forms.

Note! By changing a variable, one can also solve the problems of *predictive FIR filtering* and *smoothing FIR filtering*



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3086-3096, 2010.

# 1.1. IIR (Kalman) and FIR (Kalman-like) Strategies for State Estimation






Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 1.2. Kalman and Most Recognized Kalman-like Filters

## IIR

- 1960–1961 — (Kalman) Linear, nonstationary, Gaussian
- 1964 — (Cox and others) Extended Kalman filter, Nonlinear model
- 1975 — (Masreliez) Robust Kalman filter, Non-Gaussian noise, heavy-tailed, outliers
- 1994 — (Evensen) Ensemble Kalman filter, High dimensional systems
- 1997 — (Julier, Uhlmann) Unscented Kalman filter, Highly nonlinear model
- 2005 — (Lange) Fast Kalman filter, Models with sparse matrices
- 2009 — (Bonnabel) Invariant Extended Kalman filter, Nonlinear symmetric systems

**A part of the Estimation Theory and Theory of Optimal Control**

## FIR

- 1999 — (Kwon, Kim, Park) Receding horizon Kalman filter, Iterative
- 2002 — (Han, Kwon, Kim) Iterative FIR filter, Deterministic model, white Gaussian noise
- 2010 — (Shmaliy) Iterative Kalman-like estimator, Time-invariant linear model

**Receding Horizon Control, Further development**

# 2. Time-Varying Batch Unbiased FIR Estimator

## 2.1. State-Space Signal Model

Consider a class of *discrete time-varying linear state-space models* represented with the state and observation equations, respectively,

$$\boxed{\begin{aligned} \mathbf{x}_n &= \mathbf{A}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{w}_n\,, \\ \mathbf{y}_n &= \mathbf{C}_n \mathbf{x}_n + \mathbf{D}_n \mathbf{v}_n\,, \end{aligned}} \qquad \begin{aligned} (1) \\ (2) \end{aligned}$$

where $\mathbf{x}_n \in \Re^K$, $\mathbf{y}_n \in \Re^M$, $\mathbf{A}_n \in \Re^{K\times K}$, $\mathbf{B}_n \in \Re^{K\times P}$, $\mathbf{C}_n \in \Re^{M\times K}$, and $\mathbf{D}_n \in \Re^{M\times M}$.

$\mathbf{w}_n \in \Re^P$ and $\mathbf{v}_n \in \Re^M$ have zero mean components, $E\{\mathbf{w}_n\} = \mathbf{0}$, $E\{\mathbf{v}_n\} = \mathbf{0}$, $E\{\mathbf{w}_i \mathbf{v}_j^T\} = \mathbf{0}$, with arbitrary distributions and known covariances

$$\boxed{\mathbf{Q}_w(i,j) = E\{\mathbf{w}_i \mathbf{w}_j^T\}\,,} \quad \boxed{\mathbf{Q}_v(i,j) = E\{\mathbf{v}_i \mathbf{v}_j^T\}}$$



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 2.2. Problem Formulation

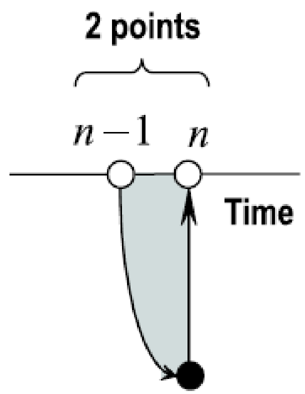
Given (1) and (2), we would like to derive a general $p$-shift Kalman-like FIR estimator for filtering ($p = 0$), prediction ($p > 0$), and smoothing ($p < 0$) of *discrete time-varying state space models* with no requirements for *noise* and *initial conditions*.

The estimator must be unbiased.

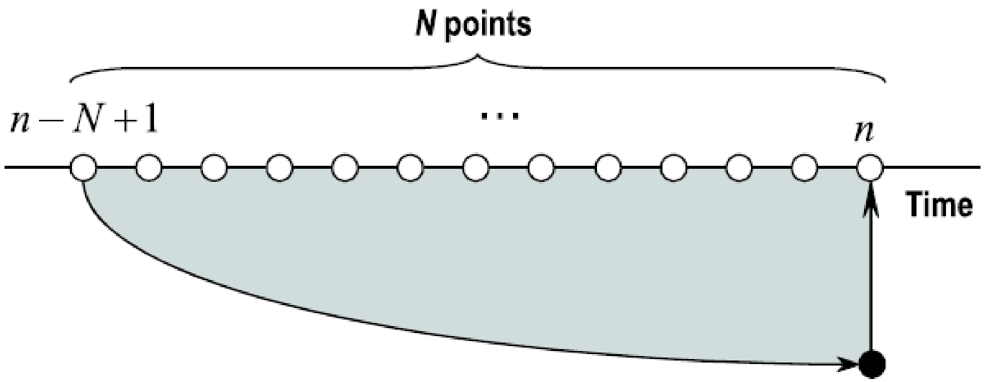
# 2.3. Main Idea and the Model Transformation

Start with the model

$$
\begin{aligned}
\mathbf{x}_n &= \mathbf{A}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{w}_n \,, \\
\mathbf{y}_n &= \mathbf{C}_n \mathbf{x}_n + \mathbf{D}_n \mathbf{v}_n \,,
\end{aligned}
$$




Expand the model on an *averaging horizon*, from $m = n - N + 1$ to $n$ *as*




Go to the *expanded state space model*

$$\boxed{\begin{array}{c} ? \\ ? \end{array}}$$

Further employ the *convolution* and find the $p$-shift estimate at $n+p$ in the minimum MSE sense, thereby solve optimally the problems of *filtering* ($p$=0), *prediction* ($p$>0), and *smoothing* ($p$<0).


Y.S. Shmaliy, *IEEE Signal Process. Letters*, vol. 15, pp. 517-520, 2008.
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3086-3096, 2010.

# 2.4. Expanded State Space Model

Using the recursively computed *forward-in-time solutions*, represent the model, (1) and (2), on a horizon of $N$ past points from $m = n - N + 1$ to $n$ as

$$\begin{array}{rcl} \mathbf{x}_n &=& \mathbf{A}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{w}_n \,, \\ \mathbf{y}_n &=& \mathbf{C}_n \mathbf{x}_n + \mathbf{D}_n \mathbf{v}_n \,, \end{array}$$

$$\begin{array}{rcl} \mathbf{X}_{n,m} &=& \mathbf{A}_{n,m} \mathbf{x}_m + \mathbf{B}_{n,m} \mathbf{W}_{n,m} \,, \\ \mathbf{Y}_{n,m} &=& \mathbf{C}_{n,m} \mathbf{x}_m + \mathbf{G}_{n,m} \mathbf{W}_{n,m} + \mathbf{D}_{n,m} \mathbf{V}_{n,m} \,, \end{array}$$

$$\mathbf{X}_{n,m} \in \Re^{KN}, \ \mathbf{Y}_{n,m} \in \Re^{MN}, \ \mathbf{W}_{n,m} \in \Re^{PN}, \ \mathbf{V}_{n,m} \in \Re^{MN} ,$$

$$\mathbf{X}_{n,m} = \left[\mathbf{x}_n^T \ \mathbf{x}_{n-1}^T \ \ldots \ \mathbf{x}_m^T\right]^T ,$$
$$\mathbf{Y}_{n,m} = \left[\mathbf{y}_n^T \ \mathbf{y}_{n-1}^T \ \ldots \ \mathbf{y}_m^T\right]^T ,$$

$$\mathbf{W}_{n,m} = \left[\mathbf{w}_n^T \ \mathbf{w}_{n-1}^T \ \ldots \ \mathbf{w}_m^T\right]^T ,$$
$$\mathbf{V}_{n,m} = \left[\mathbf{v}_n^T \ \mathbf{v}_{n-1}^T \ \ldots \ \mathbf{v}_m^T\right]^T ,$$

$$\mathbf{A}_{n,m} \in \Re^{KN\times K}, \ \mathbf{C}_{n,m} \in \Re^{MN\times K}, \ \mathbf{G}_{n,m} \in \Re^{MN\times PN},$$

$$\mathbf{D}_{n,m} \in \Re^{MN\times MN} \qquad \mathbf{B}_{n,m} \in \Re^{KN\times PN}$$

$$\mathbf{C}_{n,m} = \bar{\mathbf{C}}_{n,m}\mathbf{A}_{n,m}$$

$$\mathbf{G}_{n,m} = \bar{\mathbf{C}}_{n,m}\mathbf{B}_{n,m} ,$$

$$\mathbf{D}_{n,m} = \mathrm{diag}\big(\underbrace{\mathbf{D}_n \ \mathbf{D}_{n-1} \ \ldots \ \mathbf{D}_m}_{N}\big) ,$$

$$\bar{\mathbf{C}}_{n,m} = \mathrm{diag}\big(\underbrace{\mathbf{C}_n \ \mathbf{C}_{n-1} \ \ldots \ \mathbf{C}_m}_{N}\big) ,$$



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

$$\mathbf{A}_{n,m} = \left[ \mathcal{A}_{n,0}^{m+1\,T} \; \mathcal{A}_{n,1}^{m+1\,T} \; \ldots \; \mathbf{A}_{m+1}^{T} \; \mathbf{I} \right]^T ,$$

$$\mathcal{A}_{r,h}^{r-g} = \prod_{i=h}^{g} \mathbf{A}_{r-i} ,$$

$$\mathbf{B}_{n,m} = \begin{bmatrix} \mathbf{B}_n & \mathbf{A}_n\mathbf{B}_{n-1} & \ldots & \mathcal{A}_{n,0}^{m+2}\mathbf{B}_{m+1} & \mathcal{A}_{n,0}^{m+1}\mathbf{B}_m \\ \mathbf{0} & \mathbf{B}_{n-1} & \ldots & \mathcal{A}_{n,1}^{m+2}\mathbf{B}_{m+2} & \mathcal{A}_{n,1}^{m+1}\mathbf{B}_{m+1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{B}_{m+1} & \mathbf{A}_{m+1}\mathbf{B}_m \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{B}_m \end{bmatrix} ,$$



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 2.5. Batch Unbiased FIR Estimator

By the convolution, the estimate $\tilde{\mathbf{x}}_{n+p|n}$ of $\mathbf{x}_n$ can be found if we assign a $K \times MN$ gain matrix $\mathbf{H}_{n,m}(p)$ and claim that

$$\boxed{\begin{aligned}\tilde{\mathbf{x}}_{n+p|n} &= \mathbf{H}_{n,m}(p)\mathbf{Y}_{n,m} \\ &= \mathbf{H}_{n,m}(p)(\mathbf{C}_{n,m}\mathbf{x}_m + \mathbf{G}_{n,m}\mathbf{W}_{n,m} \\ &\quad +\mathbf{D}_{n,m}\mathbf{V}_{n,m})\,.\end{aligned}}$$

This estimate will be unbiased if the following *unbiasedness condition* is obeyed:

$$\boxed{E\{\tilde{\mathbf{x}}_{n+p|n}\} = E\{\mathbf{x}_{n+p}\}}$$

where $E(x)$ means an average of $x$.



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

By the averaging, we have

$$\boxed{E\{\tilde{\mathbf{x}}_{n+p|n}\} = \bar{\mathbf{H}}_{n,m}(p)\mathbf{C}_{n,m}\mathbf{x}_m,}$$

where $\bar{\mathbf{H}}_{n,m}(p)$ is the unbiased gain matrix.

In turn, $E\{\mathbf{x}_n\}$ can be substituted with the *p*-shift first vector row of (1) as

$$\boxed{E\{\mathbf{x}_{n+p}\} = \mathcal{A}_{n+p,0}^{m+1}\mathbf{x}_m\ .}$$

Equating $E\{\tilde{\mathbf{x}}_{n+p|n}\}$ to $E\{\mathbf{x}_{n+p}\}$ leads to the *unbiasedness constraint*

$$\mathcal{A}_{n+p,0}^{m+1} = \bar{\mathbf{H}}_{n,m}(p)\mathbf{C}_{n,m}$$

The solution gives us the *unbiased gain*

$$\boxed{\bar{\mathbf{H}}_{n,m}(p) = \mathcal{A}_{n+p,0}^{m+1}(\mathbf{C}_{n,m}^T\mathbf{C}_{n,m})^{-1}\mathbf{C}_{n,m}^T}$$



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# Theorem 1

*Theorem 1:* Given (1) and (2) with zero mean mutually uncorrelated and independent $\mathbf{w}_n$ and $\mathbf{v}_n$ having arbitrary distributions and known covariance functions. Then, filtering ($p = 0$), $p$-lag smoothing ($p < 0$), and $p$-step prediction ($p > 0$) are provided at $n+p$ using data taken from $m = n-N+1$ to $n$ by the batch FIR UE as

$$\boxed{\begin{aligned} \bar{\mathbf{x}}_{n+p|n} &= \bar{\mathbf{H}}_{n,m}(p)\mathbf{Y}_{n,m} \\ &= \mathcal{A}_{n+p,0}^{m+1}(\mathbf{C}_{n,m}^T\mathbf{C}_{n,m})^{-1}\mathbf{C}_{n,m}^T\mathbf{Y}_{n,m}\,, \end{aligned}}$$

where $\mathbf{C}_{n,m}$ is given by (12) and $\mathbf{Y}_{n,m}$ is the data vector (8).



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 3. Time-varying Kalman-like Estimator Ignoring Noise and Initial Conditions

The time-varying batch unbiased FIR estimator established by Theorem 1 has the following advantages against the Kalman filter:

- It *ignores noise and the initial conditions*, thus has strong engineering features.
- Its estimate *converges to the optimal* one when $N >> 1$ or the mean square initial state dominates the noise matrices in the order of magnitudes.

and the following disadvantages:

- Its batch algorithm implies the *computational problem*, especially when $N$ is large.

The following theorem suggests a fast iterative Kalman-like form for this estimator

# 3.1. Theorem 2

*Theorem 2:* Given the batch FIR UE (theorem 1). Then its iterative Kalman-like form is the following:

$$\boxed{\begin{aligned}\bar{\mathbf{x}}_{l+p|l} &= \mathbf{A}_{l+p}\bar{\mathbf{x}}_{l+p-1|l-1} + \mathbf{A}_{l+p}\boldsymbol{\Upsilon}_l^{-1}(p)\mathbf{F}_l\mathbf{C}_l^T \\ &\quad \times\left[\mathbf{y}_l - \mathbf{C}_l\boldsymbol{\Upsilon}_l(p)\bar{\mathbf{x}}_{l+p-1|l-1}\right],\end{aligned}}$$

$$\begin{aligned}\mathbf{F}_l &= [\mathbf{C}_l^T\mathbf{C}_l + (\mathbf{A}_l\mathbf{F}_{l-1}\mathbf{A}_l^T)^{-1}]^{-1}, \\ \bar{\mathbf{x}}_{s+p|s} &= \mathcal{A}_{s+p,0}^{m+1}\mathbf{P}\mathbf{C}_{s,m}^T\mathbf{Y}_{s,m}, \\ \mathbf{F}_s &= \mathcal{A}_{s,0}^{m+1}\mathbf{P}\mathcal{A}_{s,0}^{m+1^T}, \\ \mathbf{P} &= (\mathbf{C}_{s,m}^T\mathbf{C}_{s,m})^{-1},\end{aligned}$$

$$\boldsymbol{\Upsilon}_l(p) = \begin{cases}\mathcal{A}_{l,0}^{l-|p|}, & p<0 \quad \text{(smoothing)} \\ \mathbf{A}_l, & p=0 \quad \text{(filtering)} \\ \mathbf{I}, & p=1 \quad \text{(prediction)} \\ \prod_{i=1}^{p-1}\mathbf{A}_{l-i}^{-1}, & p>1 \quad \text{(prediction).}\end{cases}$$

where $s = l-1$ and an iterative variable $l$ ranges from $\max(m+K, m+2, m+2-p)$ to $n$ in order for $\mathbf{P}^{-1}$ to be nonsingular and $\mathcal{A}_{l-1+p,0}^{m+1}$ and $\mathcal{A}_{l-1,0}^{m+1}$ to exist. The true estimate corresponds to $l = n$.



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 3.2. Full-Horizon Kalman-Like Estimator

is used to process at once all the data available

TABLE I
FULL-HORIZON TV KALMAN-LIKE FIR UE ALGORITHM

| Stage | |
|---|---|
| Given: | $K, p, n \geqslant \alpha,\ \alpha = \max(K, 2, 2-p)$ |
| Set: | $\boldsymbol{\Upsilon}_{\alpha}(p)$ by (I.9) |
| | $\mathbf{P} = (\mathbf{C}^T_{\alpha-1,0}\mathbf{C}_{\alpha-1,0})^{-1}$ |
| | $\mathbf{F}_{\alpha-1} = \mathcal{A}^1_{\alpha-1,0}\mathbf{P}\mathcal{A}^{1^T}_{\alpha-1}$ |
| | $\bar{\mathbf{x}}_{\alpha+p-1\vert\alpha-1} = \mathcal{A}^1_{\alpha+p,1}\mathbf{P}\mathbf{C}^T_{\alpha-1,0}\mathbf{Y}_{\alpha-1,0}$ |
| Update: | $\mathbf{F}_n = [\mathbf{C}^T_n\mathbf{C}_n + (\mathbf{A}_n\mathbf{F}_{n-1}\mathbf{A}^T_n)^{-1}]^{-1}$ |
| | $\bar{\mathbf{x}}_{n+p\vert n} = \mathbf{A}_{n+p}\bar{\mathbf{x}}_{n+p-1\vert n-1} + \mathbf{A}_{n+p}\boldsymbol{\Upsilon}^{-1}_n(p)\mathbf{F}_n\mathbf{C}^T_n$ |
| | $\times[\mathbf{y}_n - \mathbf{C}_n\boldsymbol{\Upsilon}_n(p)\bar{\mathbf{x}}_{n+p-1\vert n-1}]$ |



Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 3.3. Noise Power Gain

The NPG $\mathbf{K}_k \triangleq \mathbf{K}_k(n, N, p)$ can be computed for the $k$th state measured with noise having $\sigma_k^2$ as

$$\mathbf{K}_k = \mathcal{H}_k \mathcal{H}_k^T = \begin{bmatrix} K_{k(11)} & \cdots & K_{k(1k)} & \cdots & K_{k(1K)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{k(k1)} & \cdots & K_{k(kk)} & \cdots & K_{k(kK)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ K_{k(K1)} & \cdots & K_{k(Kk)} & \cdots & K_{k(KK)} \end{bmatrix}$$

where the thinned $K \times N$ gain $\mathcal{H}_k \triangleq \left(\bar{\mathbf{H}}_{n,m}(p)\right)_k$ is composed by $K$th columns of $\bar{\mathbf{H}}_{n,m}(p)$ starting with the $k$th one.



Y.S. Shmaliy, O. Ibarra-Manzano, *IEEE Signal Process. Letters*, vol. 18, no. 4, pp. 207-210, Apr. 2011

# 3.4. Estimate Error Bound

Setting aside the rigorous error matrix, the estimate *error bound* (EB) can easily be ascertained via the NPG matrix $\mathbf{K}_k$ in the *three-sigma sense* as

$$\boxed{\beta_{k(vg)}(n, N, p) = 3\sigma_k K_{k(vg)}^{1/2}(n, N, p)} \qquad (30)$$

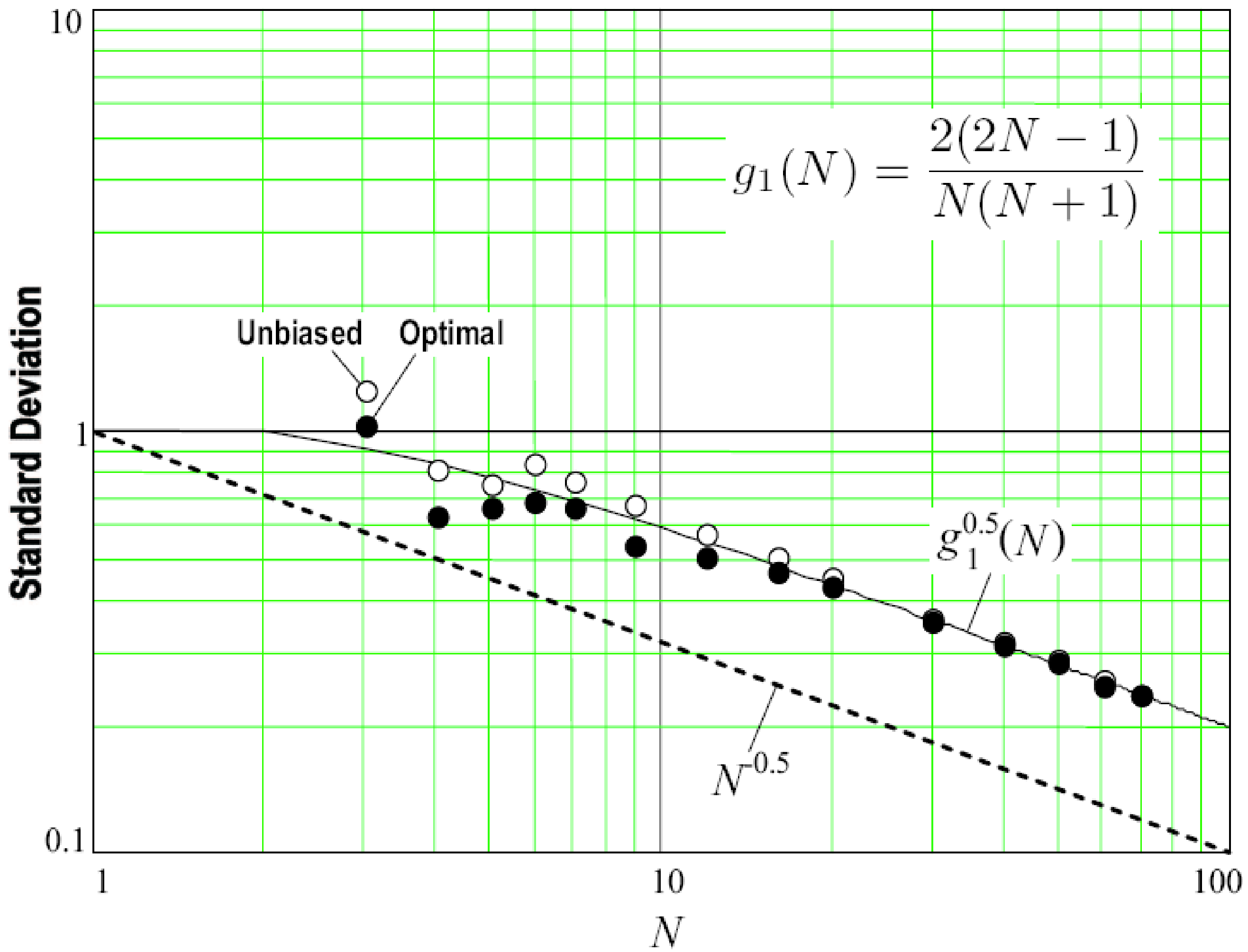
to characterize denoising in the *v*-to-*g* channel via measurement of the *k*th state.

In what follows, we shall show that EB is an *efficient measure of errors* in FIR and IIR estimators.



Y.S. Shmaliy, O. Ibarra-Manzano, *IEEE Signal Process. Letters*, vol. 18, no. 4, pp. 207-210, Apr. 2011

# 3.5. Example: Optimal vs. Unbiased FIR Estimates




As can be seen, the optimal and unbiased estimates converge with increasing $N$ and become indistinguishable when $N > 30$.


Y.S. Shmaliy, *IEEE Trans. UFFC,* vol. 53, no. 5, pp. 862-870, May 2006.
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 58, no. 6, pp. 3086-3096, 2010.

# 4.1. Filtering of a Time-Varying Polynomial Model

Consider a two-state polynomial model

$$\boxed{\begin{aligned}\mathbf{x}_n &= \mathbf{A}_n\mathbf{x}_{n-1} + \mathbf{B}_n\mathbf{w}_n\,,\\ \mathbf{y}_n &= \mathbf{C}_n\mathbf{x}_n + \mathbf{D}_n\mathbf{v}_n\,,\end{aligned}}$$

with $\mathbf{B}_n = \mathbf{I}$, $\mathbf{D}_n = \mathbf{I}$, $\mathbf{C}_n = [1\ 0]$, and

$$\mathbf{A}_n = \begin{bmatrix} 1 & (1+d_n)\tau \\ 0 & 1 \end{bmatrix},$$

where $d_n = 20$ if $120 \leqslant n \leqslant 160$ and $d_n = 0$ otherwise.

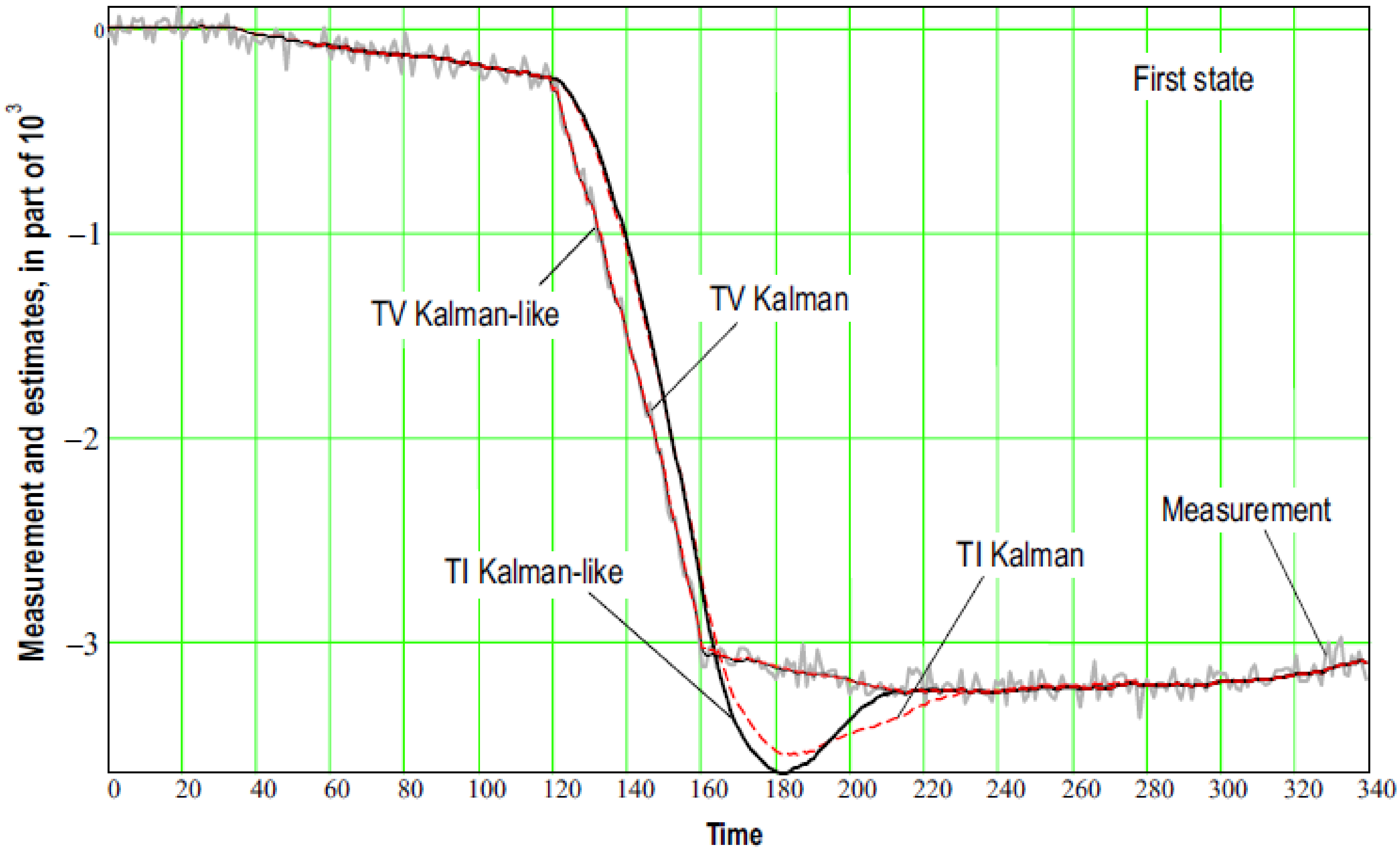
Allow $\boxed{\sigma_1^2 = 10^{-4}}$, $\boxed{\sigma_2^2 = 4\times 10^{-2}/\mathrm{s}^2}$, and $\boxed{\sigma_v^2 = 50^2}$. in both the model and the filters.

Provide Kalman and Kalman-like FIR filtering and compare the estimates.



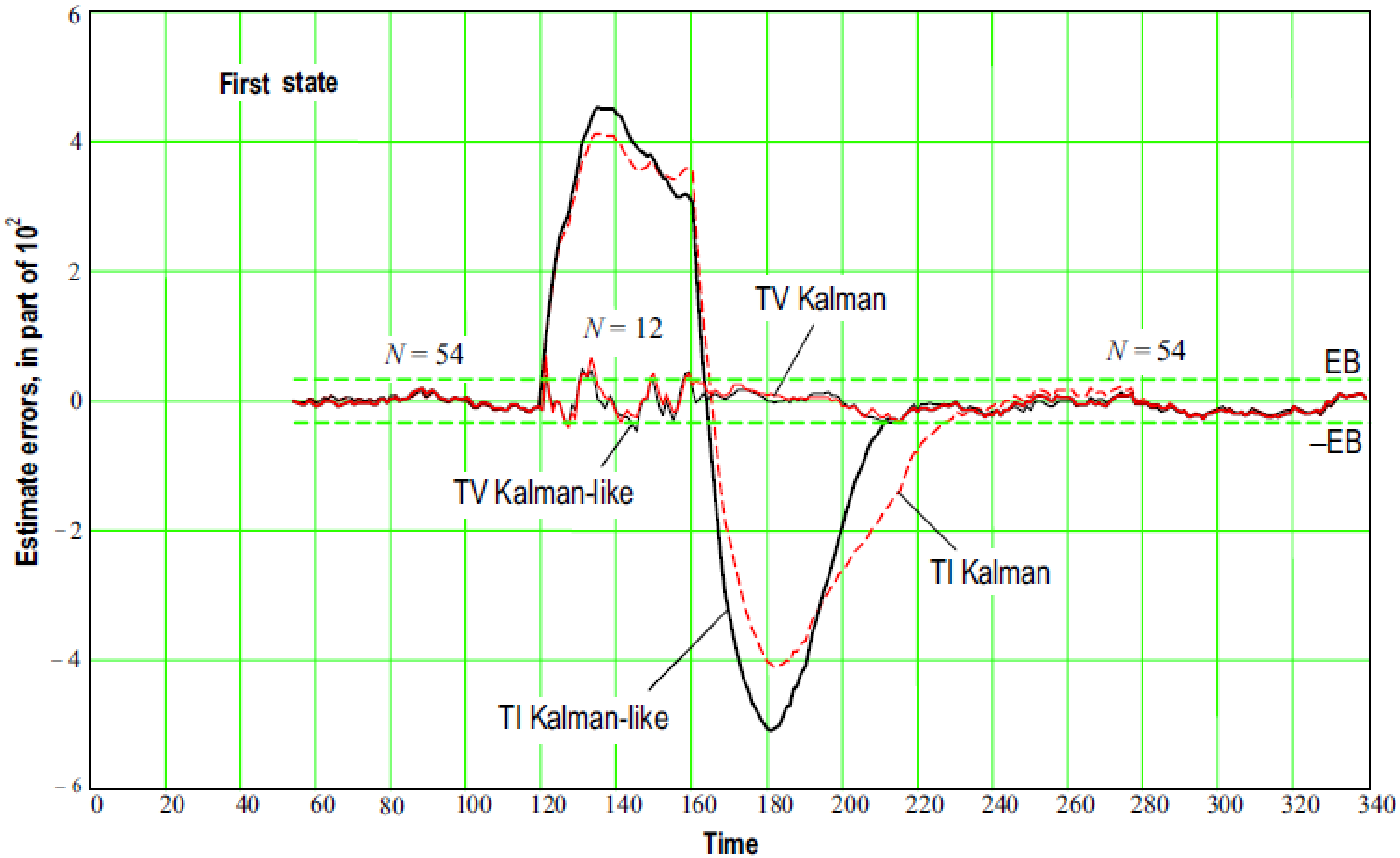
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 4.1.1. Time-Varying (TV) and Time-Invariant (TI) Filtering of the First State a Two-State Polynomial Model



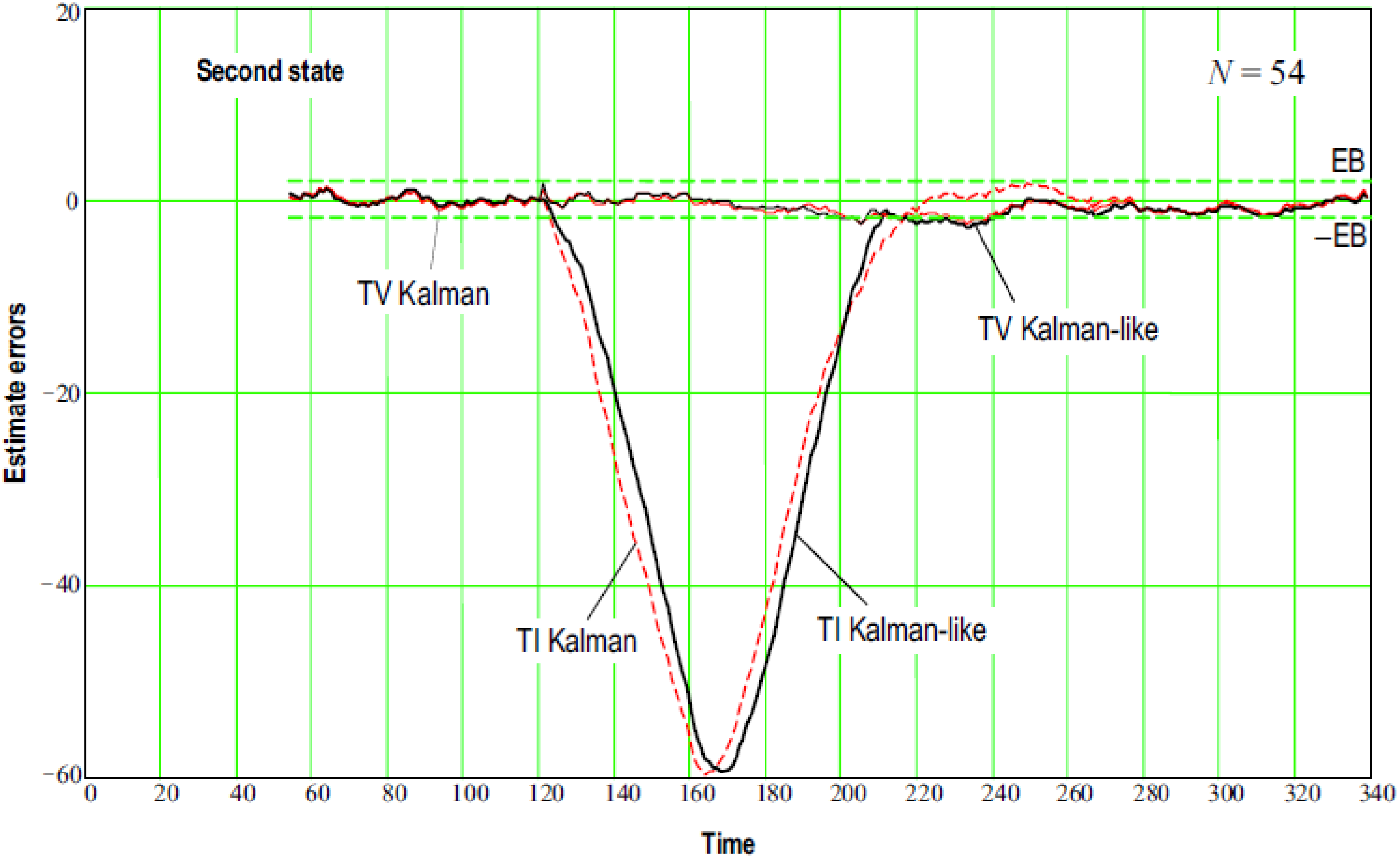
(a)

# 4.1.2. Estimate Errors in the First State




(b)

# 4.1.3. Estimate Errors in the Second State




(c)

# 4.2. Filtering with Errors in Noise Covariances

Consider a two-state polynomial model

$$\boxed{\begin{aligned} \mathbf{x}_n &= \mathbf{A}_n \mathbf{x}_{n-1} + \mathbf{B}_n \mathbf{w}_n\,, \\ \mathbf{y}_n &= \mathbf{C}_n \mathbf{x}_n + \mathbf{D}_n \mathbf{v}_n\,, \end{aligned}}$$

with $\mathbf{B}_n = \mathbf{I}$, $\mathbf{D}_n = \mathbf{I}$, $\mathbf{C}_n = [1\ 0]$, and

$$\mathbf{A}_n = \begin{bmatrix} 1 & (1+d_n)\tau \\ 0 & 1 \end{bmatrix},$$

where $d_n = 20$ if $120 \leqslant n \leqslant 160$ and $d_n = 0$ otherwise.

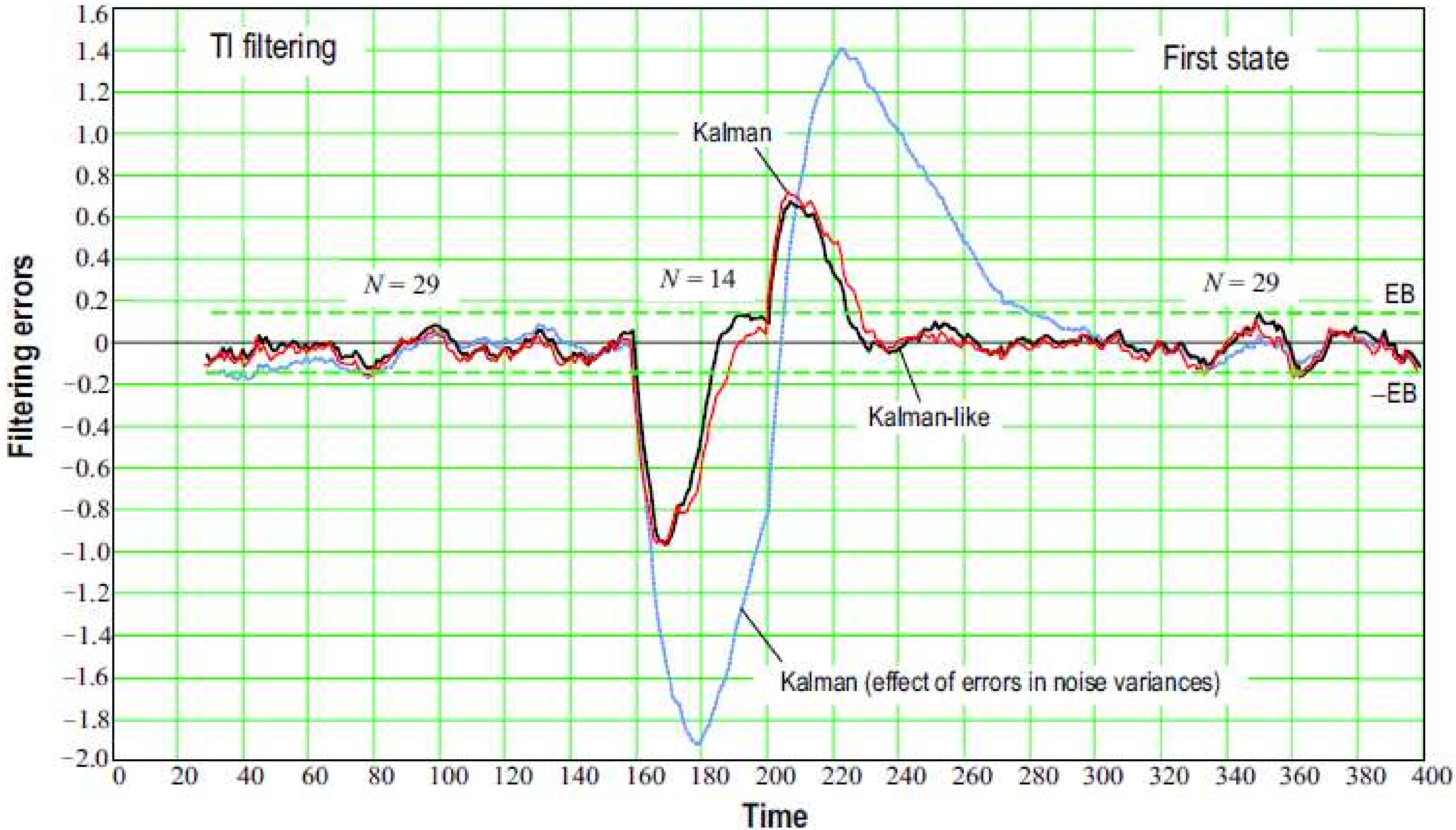
Because the system noise is often hard to describe exactly, allow errors in the noise covariances as $\boxed{\sigma_1^2 = 0.25 \times 10^{-4}}$ against actual $\boxed{\sigma_1^2 = 10^{-4}}$ and $\boxed{\sigma_2^2 = 0.25 \times 10^{-6}/\ \mathrm{s}}$ against actual $\boxed{\sigma_2^2 = 4 \times 10^{-6}/\mathrm{s}^2}$ .

Provide Kalman and Kalman-like FIR filtering and compare the estimates.



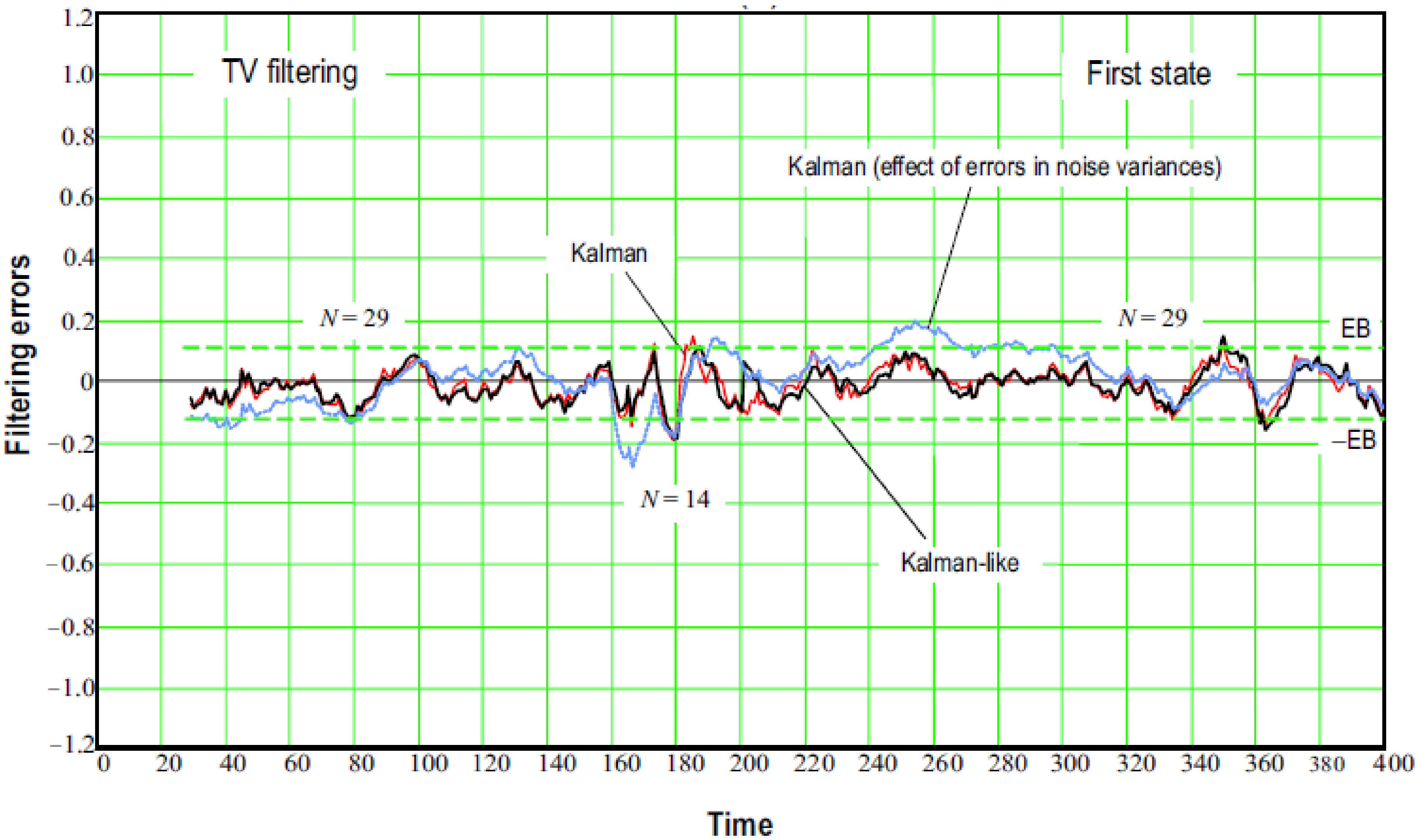
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 4.2.1. Estimate Errors in TI Filtering of the First State



(a)

# 4.2.2. Estimate Errors in the TV Filtering of the Second State




(b)

# 4.3. Prediction of the Two-State Model

Consider a two-state polynomial model with $\mathbf{B}_n = \mathbf{I}$, $\mathbf{D}_n = \mathbf{I}$, $\mathbf{C}_n = [\,1\ 0\,]$,
and

$$\mathbf{A}_n = \begin{bmatrix} 1 & (1+d_n)\tau \\ 0 & 1 \end{bmatrix},$$

with $x_{10} = 1$, $x_{20} = 0.01/\mathrm{s}$, $\sigma_1^2 = 10^{-6}$, $\sigma_2^2 = 10^{-4}/\mathrm{s}^2$
and $d_n = 20$ from 150 to 152.

For the Kalman filter, allow errors in the initial states as
$x_{10} = 2$ and $x_{20} = 0.03/\mathrm{s}$.

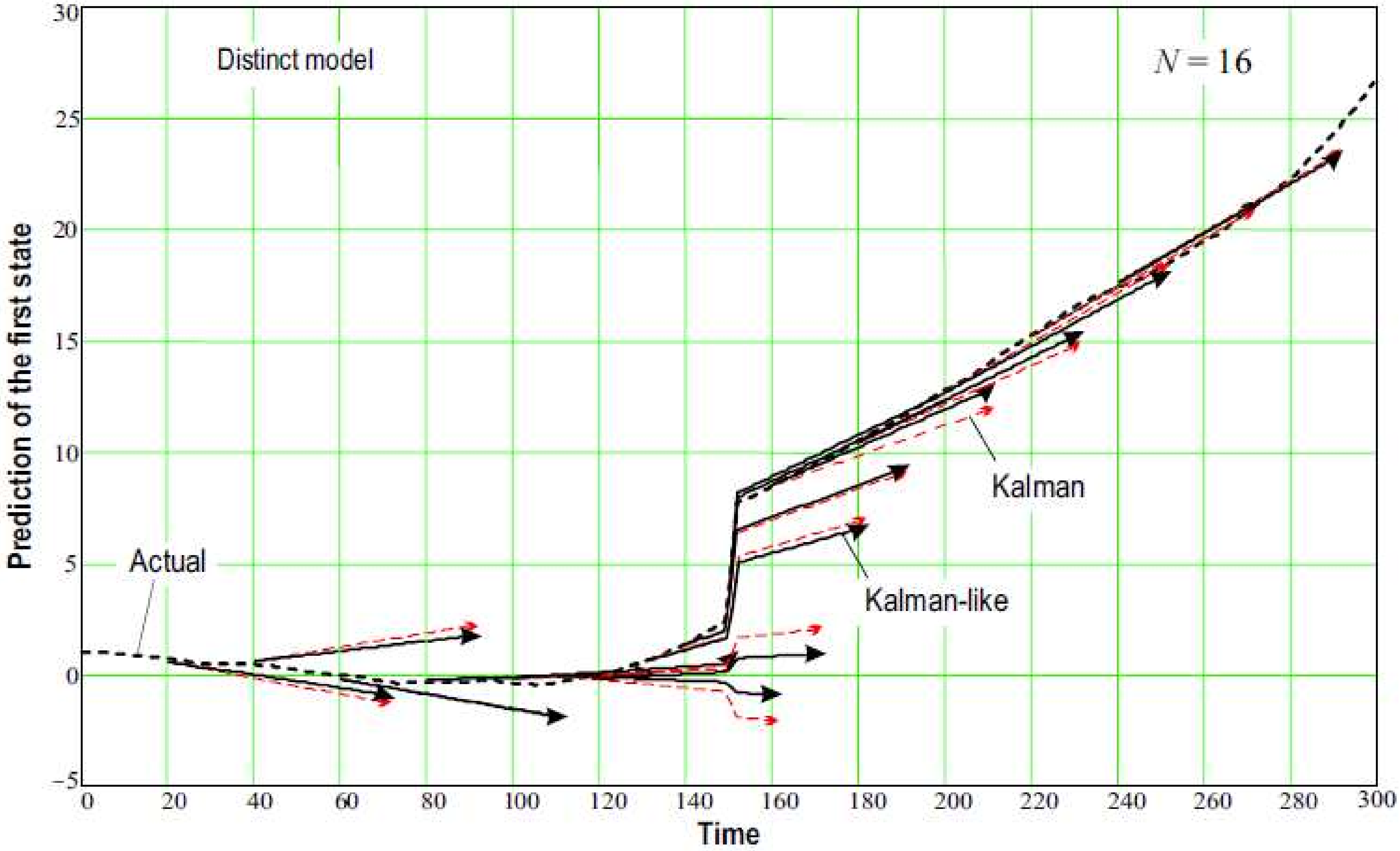
Provide prediction of the model behavior employing the Kalman and Kalman-like FIR predictors. Organize the Kalman prediction as

$$\boxed{\tilde{\mathbf{x}}_{n+p|n} = \prod_{i=1}^{p} \mathbf{A}_{n+p+1-i}\bar{\mathbf{x}}_{n|n}\,.}$$



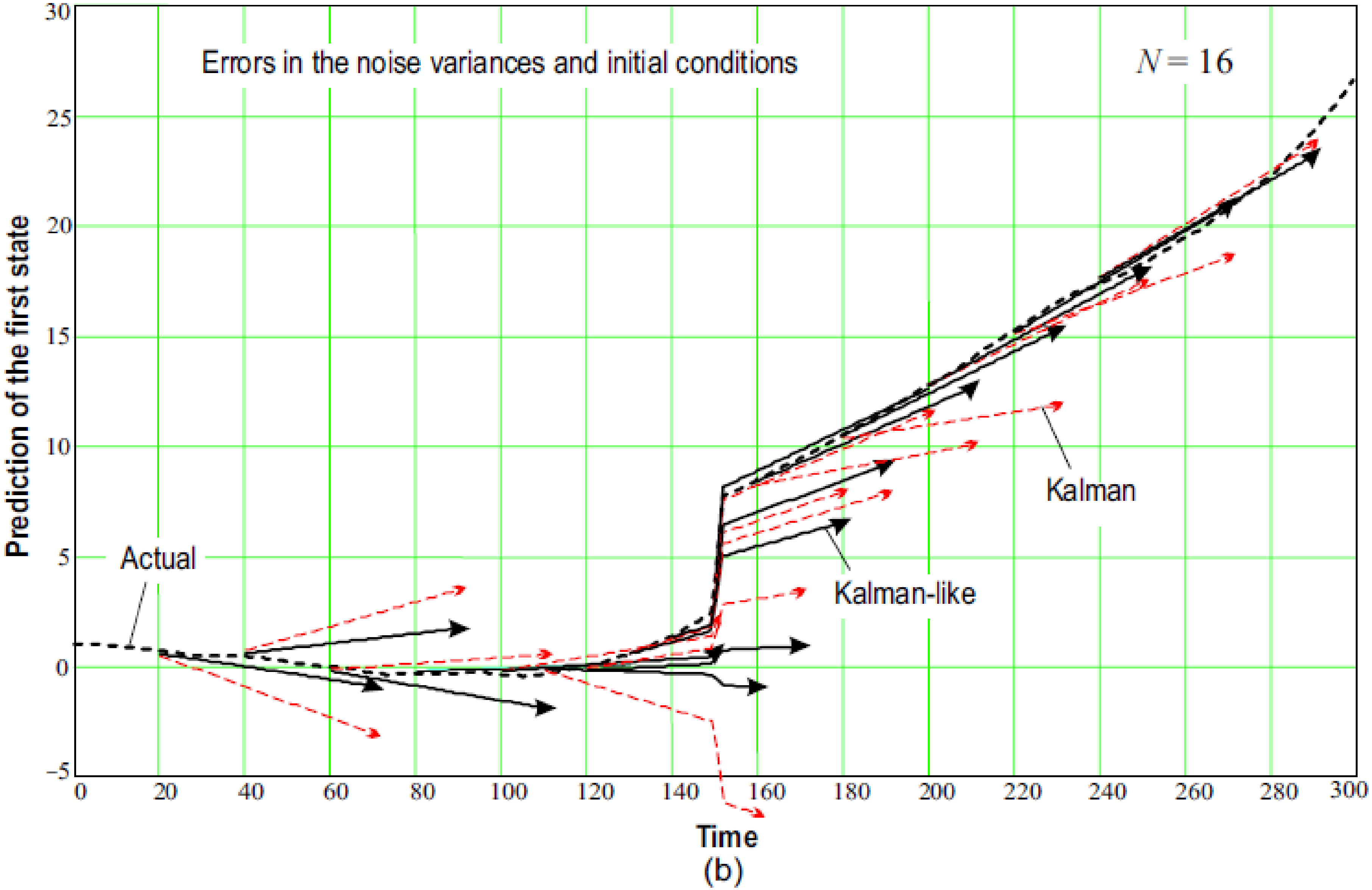
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

# 4.3.1. Prediction of a Distinct Model




(a)

# 4.3.2. Prediction with Error in the Model Description



# 4.4. Filtering of a Gaussian Model with Outliers

Consider a two-state polynomial model with $\mathbf{B}_n = \mathbf{I}$, $\mathbf{D}_n = \mathbf{I}$, $\mathbf{C}_n = [1\ 0]$, and

$$\mathbf{A}_n = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}$$

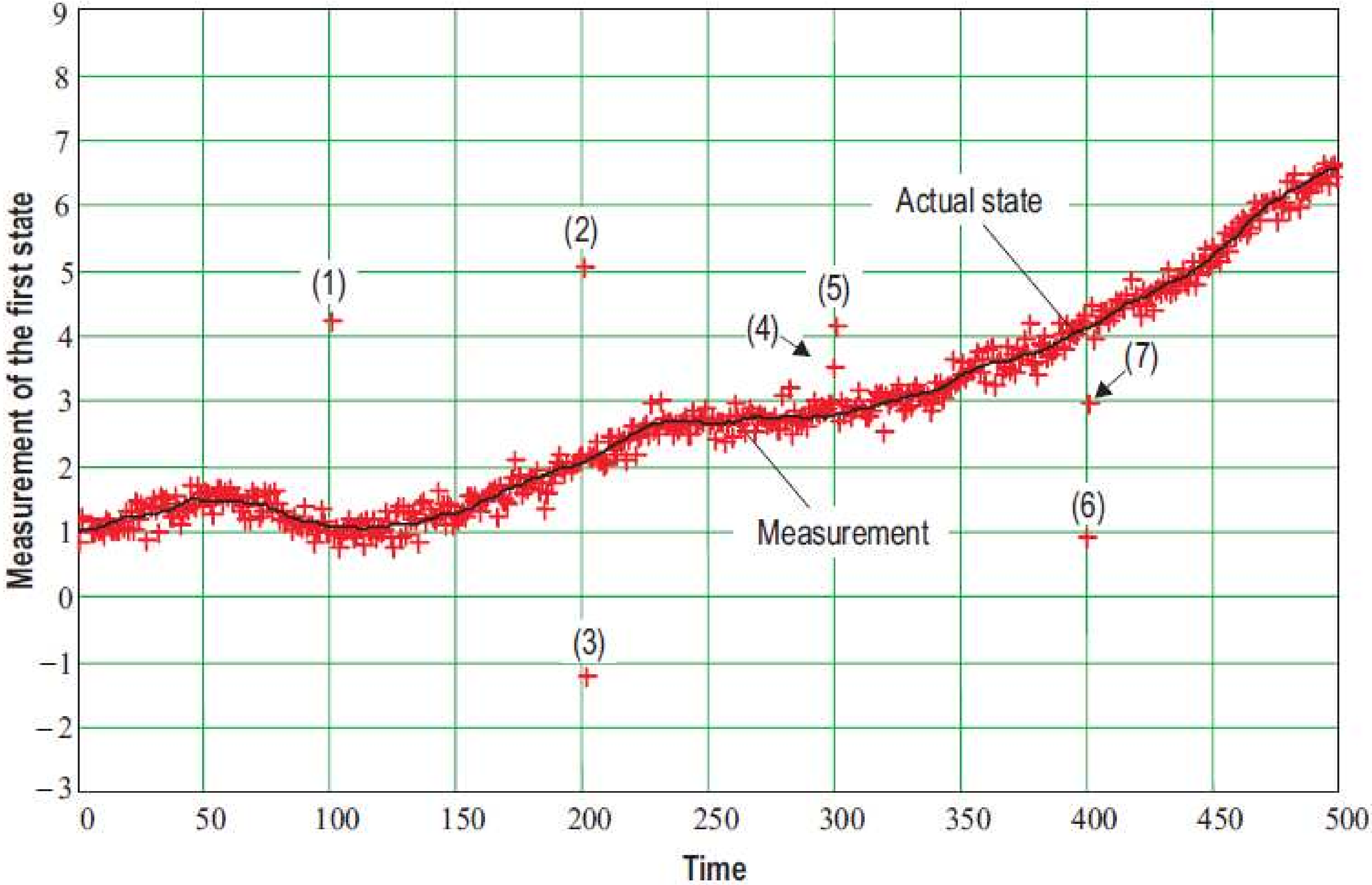
with $x_{10} = 1$, $x_{20} = 0.01/\mathrm{s}$, $\sigma_1^2 = 10^{-6}$, $\sigma_2^2 = 10^{-4}/\mathrm{s}^2$ and $\sigma_v^2 = 0.0225$. For the Kalman filter, increase $\sigma_1$ and $\sigma_2$ by the factors of 2 and 3, respectively.

Induce outliers to measurement and provide estimation employing the Kalman and Kalman-like FIR filters.



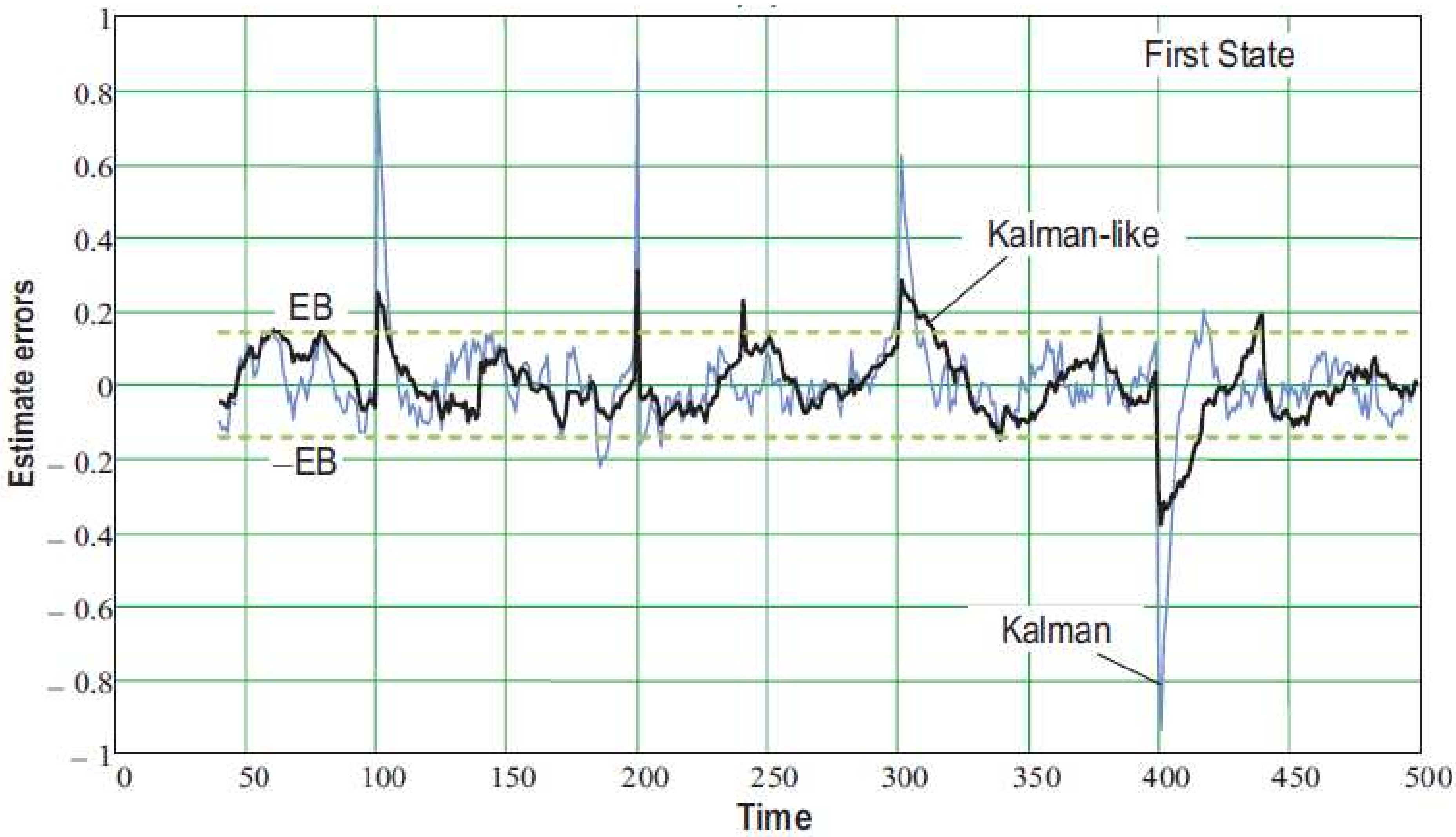
Y.S. Shmaliy, *IEEE Trans. Signal Process.*, vol. 59, no. 6, pp. 2465-2473, 2011.

## 4.4.1. Measurement of a Gaussian Model with Outliers



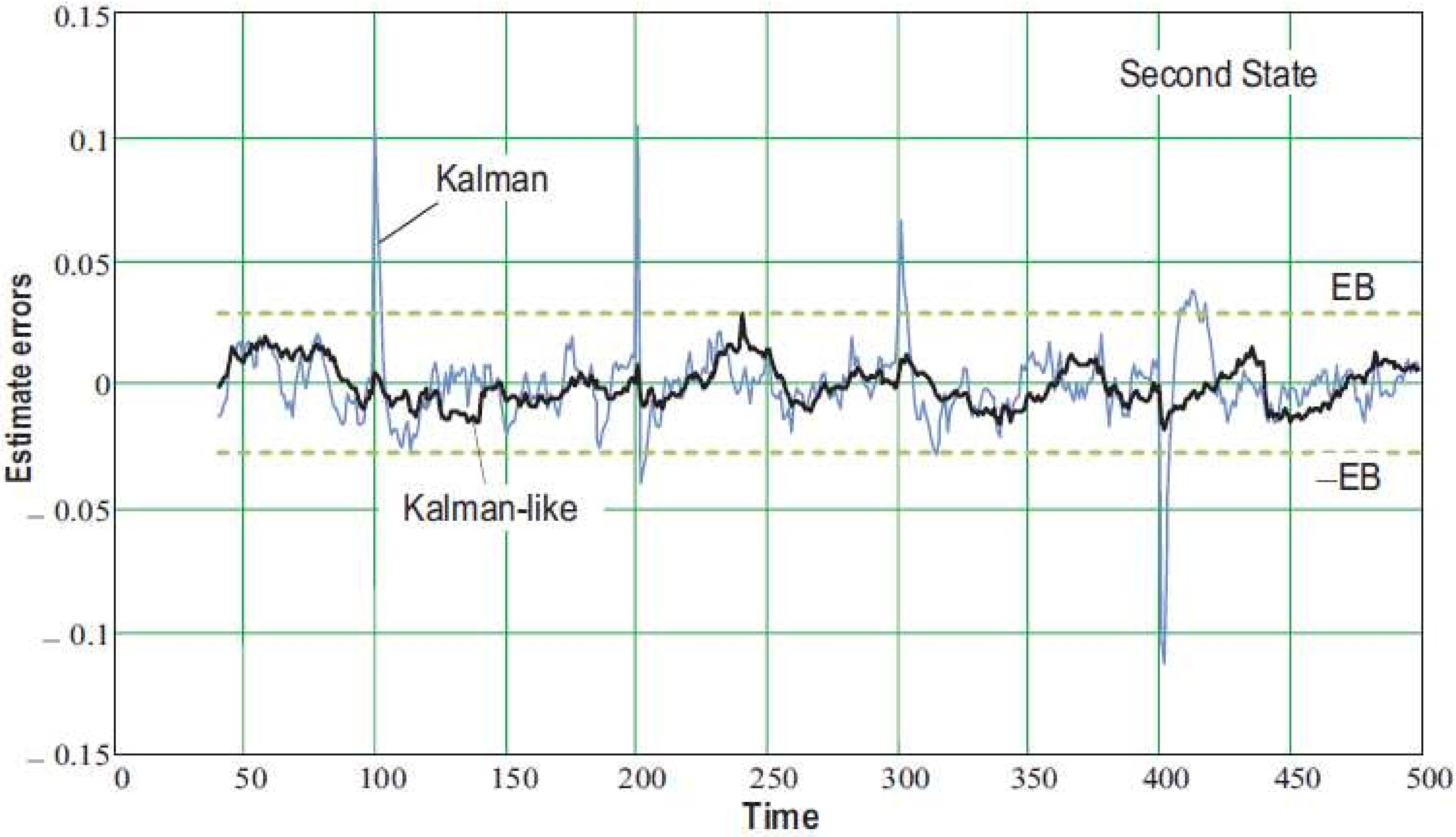

(a)

## 4.4.2. Estimate Errors in the First State



(b)

## 4.4.3. Estimate Errors in the Second State



(c)

# Conclusions:

- *The Kalman-like unbiased FIR estimator ignoring noise and initial conditions* is a nice tool for near optimum estimation and denoising. The estimator outperforms the *Kalman filter* if
  - *Noise and initial conditions are not known exactly*.
  - Both the *system and measurement noise components need to be filtered out.*
  - Models are measured in the *non-Gaussian*, *heavy tailed*, and *Gaussian with outliers* noise environments.
  - Models have *temporary uncertainties*.
- The payment of about *N times larger computational time* required by averaging will not be necessary in parallel computing.