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A Lagrangian Dual Relaxation Approach to ML MIMO Detection: Reinterpreting Regularized Lattice Decoding

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Outline

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MIMO Problem

Complex-valued $M_C \times N_C$ MIMO signal model:

$$\boldsymbol{y}_C = \boldsymbol{H}_C \boldsymbol{s}_C + \boldsymbol{\nu}_C$$

where

received signal:	$\boldsymbol{y}_C \in \mathbb{C}^{M_C}$;
channel:	$\boldsymbol{H}_C \in \mathbb{C}^{M_C \times N_C}$;
AWGN:	$\boldsymbol{\nu}_C \in \mathbb{C}^{M_C}$;
transmitted symbols:	$\boldsymbol{s}_C \in \mathcal{A}^{N_C}$;
$(u+1)^2$-QAM constellation:	$\mathcal{A} = \{s_R + js_I \mid s_R, s_I \in \{\pm 1, \pm 3, \ldots, \pm u\}\}$.

Goal: Detect $\boldsymbol{s}_C$ from $\boldsymbol{y}_C$, given that $\boldsymbol{H}_C$ is known.

Many applications: spatial multiplexing, multiuser CDMA, and many others...

Real Model

- Complex model

$$\boldsymbol{y}_C = \boldsymbol{H}_C \boldsymbol{s}_C + \boldsymbol{\nu}_C$$

- Equivalent real model:

$$\underbrace{\begin{bmatrix} \Re\{\boldsymbol{y}_C\} \\ \Im\{\boldsymbol{y}_C\} \end{bmatrix}}_{\boldsymbol{y} \in \mathbb{R}^M} = \underbrace{\begin{bmatrix} \Re\{\boldsymbol{H}_C\} & -\Im\{\boldsymbol{H}_C\} \\ \Im\{\boldsymbol{H}_C\} & \Re\{\boldsymbol{H}_C\} \end{bmatrix}}_{\boldsymbol{H} \in \mathbb{R}^{M \times N}} \underbrace{\begin{bmatrix} \Re\{\boldsymbol{s}_C\} \\ \Im\{\boldsymbol{s}_C\} \end{bmatrix}}_{\boldsymbol{s} \in \mathbb{R}^N} + \underbrace{\begin{bmatrix} \Re\{\boldsymbol{\nu}_C\} \\ \Im\{\boldsymbol{\nu}_C\} \end{bmatrix}}_{\boldsymbol{\nu} \in \mathbb{R}^M}$$

where $M = 2M_C$, and $N = 2N_C$.

Real Model

- Complex model

$$\boldsymbol{y}_C = \boldsymbol{H}_C \boldsymbol{s}_C + \boldsymbol{\nu}_C$$

- Equivalent real model:

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{\nu}$$

- Real-valued constellation set:

$$\begin{aligned} \boldsymbol{s} &\in \{\boldsymbol{s} \in \mathbb{R}^N \mid s_i \in \{\pm 1, \pm 3, \ldots, \pm u\},\ \forall\ i\} \\ &= \{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1} \mid \boldsymbol{s}^2 \preceq u^2 \mathbf{1}\} \end{aligned}$$

The Maximum Likelihood Detection

$$\begin{aligned} \min_{\boldsymbol{s}} \quad & \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \\ \text{s.t.} \quad & \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \\ & \boldsymbol{s}^2 \preceq u^2\mathbf{1}. \end{aligned}$$

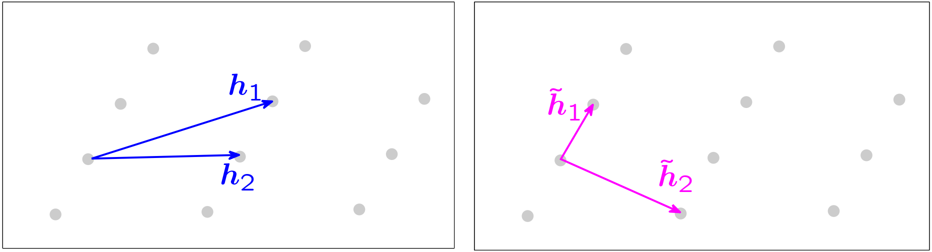
- Search for a lattice point within the symbol bound.
- Optimum, but NP-hard in general [Grotschel et al., 1993].
- Can be efficiently computed by a sphere decoder; the complexity is still exponential in N [Jaldén and Ottersten, 2005].

From ML to Naive Lattice Decoding

- To reduce complexity, lattice reduction is applied [Windpassinger et al., 2004].

$$\begin{aligned} \min_{\boldsymbol{s}} \quad & \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \\ \text{s.t.} \quad & \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \\ & \boldsymbol{s}^2 \preceq u^2\mathbf{1}. \end{aligned} \quad \Longleftrightarrow \quad \begin{aligned} \min_{\boldsymbol{s}} \quad & \|\boldsymbol{y} - \overbrace{\boldsymbol{H}\boldsymbol{U}}^{\tilde{\boldsymbol{H}}} \overbrace{\boldsymbol{U}^{-1}\boldsymbol{s}}^{\tilde{\boldsymbol{s}}}\|^2 \\ \text{s.t.} \quad & \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \\ & \boldsymbol{s}^2 \preceq u^2\mathbf{1}. \end{aligned}$$

where $\boldsymbol{U} \in \mathbb{Z}^{N\times N}$ is unimodular ($\boldsymbol{U}\boldsymbol{s} \in \mathbb{Z}^{N\times N} \Leftrightarrow \boldsymbol{s} \in \mathbb{Z}^{N\times N}$).



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- It is difficult to handle the symbol bound $\tilde{\boldsymbol{s}} \in \{\boldsymbol{U}^{-1}\boldsymbol{s} \mid \boldsymbol{s}^2 \preceq u^2\boldsymbol{1}\}$ in sphere decoding.
- The symbol bound is simply discarded in naive lattice decoding.

Naive lattice Decoder (NLD)

$$\begin{aligned} \min_{\boldsymbol{s}} \quad & \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \\ \text{s.t.} \quad & \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1} \end{aligned}$$

- Removing the symbol bounds allows lattice reduction, which reduces the complexity of sphere decoding [Murugan et al., 2006].
- Removing bound control degrades error rate performance.
- May not achieve the optimal diversity multiplexing tradeoff (DMT) [El Gamal et al., 2004] [Taherzadeh and Khandani, 2010].

Regularized Lattice Decoder

$$\min_{\boldsymbol{s}} \quad \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T\boldsymbol{T}\boldsymbol{s}$$
$$\text{s.t.} \quad \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}$$

- $\boldsymbol{T}$ is a predefined positive definite matrix.
- Regularization penalizes $\boldsymbol{s}$ that is far away from the origin.
- Achieves optimal DMT [Jaldén and Elia, 2010] [El Gamal et al., 2004].
- Lattice reduction aided (LRA) detection (e.g., lattice reduction + DF), a low complexity LD approx., also achieves optimal DMT [Jaldén and Elia, 2010].
- MMSE regularization: $\boldsymbol{T} = \frac{\sigma_\nu^2}{\sigma_s^2}\boldsymbol{I}$ [Murugan et al., 2006].

Motivation

- Regularized lattice decoding can preserve optimal DMT [Jaldén and Elia, 2010].
- We don't know other regularizations, other than the MMSE.
- Goal : Find a good regularization by a systematic approach.

Outline

1. Background
2. Lagrangian Dual Relaxation-based Lattice Decoding
   - Lagrangian Dual Formulation
   - Projected Subgradient Method
3. Simulation Results
4. Summary and Extensions

Key idea of the Lagrangian Dual Relaxation Approach

- Lagrangian dual relaxation (LDR) formulation of the ML problem
  - ◇ Treat regularized lattice decoding from a Lagrangian duality viewpoint
- Projected subgradient method
  - ◇ Adaptive regularization to control the symbol bound

Lagrangian Dual Formulation

Primal Problem:

$$
\text{(P)} \quad \begin{aligned} \min_{\boldsymbol{s}} \quad & \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \\ \text{s.t.} \quad & \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \quad \text{(Problem domain)} \\ & \boldsymbol{s}^2 \preceq u^2 \mathbf{1}. \end{aligned}
$$

Lagrangian Dual Formulation

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$$\text{(P)} \quad \min_{\boldsymbol{s}} \quad \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2$$
$$\text{s.t.} \quad \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \quad \text{(Problem domain)}$$
$$\boldsymbol{s}^2 \preceq u^2\mathbf{1}.$$

Lagrangian function with Lagrangian multiplier $\boldsymbol{\lambda} \succeq \mathbf{0}$:

$$\mathcal{L}(\boldsymbol{s}, \boldsymbol{\lambda}) = \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{\lambda}^T(\boldsymbol{s}^2 - u^2\mathbf{1})$$
$$= \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T\boldsymbol{D}(\boldsymbol{\lambda})\boldsymbol{s} - u^2\boldsymbol{\lambda}^T\mathbf{1}$$

where $\boldsymbol{D}(\boldsymbol{\lambda}) = \text{diag}(\boldsymbol{\lambda})$.

Dual function:

$$d(\boldsymbol{\lambda}) = \min_{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}} \mathcal{L}(\boldsymbol{s}, \boldsymbol{\lambda})$$

Lagrangian Dual Formulation

Primal Problem:

$$\text{(P)} \quad \min_{\boldsymbol{s}} \; \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 \quad \text{s.t.} \; \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}, \quad \text{(Problem domain)} \quad \boldsymbol{s}^2 \preceq u^2\mathbf{1}.$$

Lagrangian function with Lagrangian multiplier $\boldsymbol{\lambda} \succeq \mathbf{0}$:

$$\begin{aligned} \mathcal{L}(\boldsymbol{s}, \boldsymbol{\lambda}) &= \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{\lambda}^T(\boldsymbol{s}^2 - u^2\mathbf{1}) \\ &= \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T\boldsymbol{D}(\boldsymbol{\lambda})\boldsymbol{s} - u^2\boldsymbol{\lambda}^T\mathbf{1} \end{aligned}$$

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Dual function:

$$d(\boldsymbol{\lambda}) = \min_{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}} \mathcal{L}(\boldsymbol{s}, \boldsymbol{\lambda})$$

Lagrangian dual problem:

$$\text{(LDR)} \qquad \max_{\boldsymbol{\lambda} \succeq \mathbf{0}} \; \min_{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T\boldsymbol{D}(\boldsymbol{\lambda})\boldsymbol{s} - u^2\boldsymbol{\lambda}^T\mathbf{1}$$

LDR and Regularized Lattice Decoding

Rewrite a little bit:

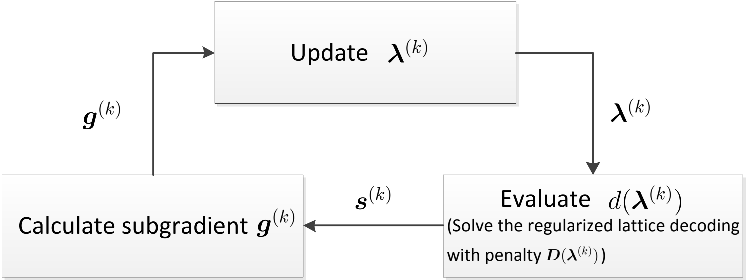
$$\text{(LDR)} \qquad \max_{\boldsymbol{\lambda}} \; d(\boldsymbol{\lambda}) = \varphi(\boldsymbol{\lambda}) - u^2 \boldsymbol{\lambda}^T \mathbf{1} \quad \text{s.t.} \; \boldsymbol{\lambda} \succeq \mathbf{0}.$$

$$(\Phi_{\boldsymbol{\lambda}}) \quad \varphi(\boldsymbol{\lambda}) = \min_{\boldsymbol{s}} \; \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T \boldsymbol{D}(\boldsymbol{\lambda}) \boldsymbol{s} \quad \text{s.t.} \; \boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}.$$

- $(\Phi_{\boldsymbol{\lambda}})$ is a diagonally regularized lattice decoding problem.
- LDR tries to find the 'best' diagonal regularization.
- Naive lattice decoding and MMSE lattice decoding can be viewed as particular instances of LDR.
- By solving LDR, a better regularization may be attained.

The Projected Subgradient Method

Block diagram for the projected subgradient method:



where $\boldsymbol{g}^{(k)}$ is a subgradient of $d(\boldsymbol{\lambda})$ at $\boldsymbol{\lambda}^{(k)}$,
$\boldsymbol{s}^{(k)}$ is the solution of $(\Phi_{\boldsymbol{\lambda}})$ for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(k)}$.

Insight: Adaptive regularization by updating $\boldsymbol{\lambda}^{(k)}$ according to $\boldsymbol{s}^{(k)}$.

The Projected Subgradient Method

- Updating $\boldsymbol{\lambda}^{(k)}$:

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{P}_{\mathbb{R}_+^N}(\boldsymbol{\lambda}^{(k)} + \alpha^{(k)}\boldsymbol{g}^{(k)})$$

where $\boldsymbol{g}^{(k)}$ is a subgradient of $d(\boldsymbol{\lambda})$ at $\boldsymbol{\lambda}^{(k)}$,
$\alpha^{(k)}$ is a predefined step size,
$\boldsymbol{P}_{\mathbb{R}_+^N}(\cdot)$ is the projection operator onto $\mathbb{R}_+^N$.

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- Computing subgradient $\boldsymbol{g}^{(k)}$:

$$\boldsymbol{g}^{(k)} = (\boldsymbol{s}^{(k)})^2 - u^2\mathbf{1}$$

where $\boldsymbol{s}^{(k)}$ is the solution of $(\Phi_{\boldsymbol{\lambda}})$ for $\boldsymbol{\lambda} = \boldsymbol{\lambda}^{(k)}$, i.e.

$$\boldsymbol{s}^{(k)} = \arg \min_{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T \boldsymbol{D}(\boldsymbol{\lambda}^{(k)})\boldsymbol{s}.$$

Adaptive Symbol Bound Control Interpretation

- Adaptive symbol bound control interpretation:

$$\begin{aligned} \boldsymbol{s}^{(k)} &= \arg \min_{\boldsymbol{s} \in 2\mathbb{Z}^N + \mathbf{1}} \|\boldsymbol{y} - \boldsymbol{H}\boldsymbol{s}\|^2 + \boldsymbol{s}^T \boldsymbol{D}(\boldsymbol{\lambda}^{(k)})\boldsymbol{s} \\ \boldsymbol{g}^{(k)} &= (\boldsymbol{s}^{(k)})^2 - u^2\mathbf{1} \\ \boldsymbol{\lambda}^{(k+1)} &= \boldsymbol{P}_{\mathbb{R}_+^N}(\boldsymbol{\lambda}^{(k)} + \alpha^{(k)}\boldsymbol{g}^{(k)}) \end{aligned}$$

  - ◇ If $(s_i^{(k)})^2 > u^2$, then $g_i^{(k)} > 0 \Rightarrow \lambda_i^{(k+1)} \uparrow$

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$$\boldsymbol{g}^{(k)} = (\boldsymbol{s}^{(k)})^2 - u^2 \mathbf{1}$$

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\boldsymbol{g}^{(k)} &= (\boldsymbol{s}^{(k)})^2 - u^2 \mathbf{1} \\
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  - ◇ If $(s_i^{(k)})^2 < u^2$, then $g_i^{(k)} < 0 \Rightarrow \lambda_i^{(k+1)} \downarrow$

- But lattice decoding is still NP-hard...

Suboptimal Lattice Decoding Solvers

Low complexity lattice decoding approximations:

- Lattice reduction aided Decision Feedback (LRA-DF) (lattice reduction + DF) [Yao and Wornell, 2002] [Windpassinger and Fischer, 2003].
- Lazy Decision Feedback (lazy-DF) (straight DF).
- or other lattice decoding approximations [Wübben et al., 2011].

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LDR Lattice Decoding

Stopping Criteria

- Maximum no. of iterations: 10
- $\|\boldsymbol{\lambda}^{(k+1)} - \boldsymbol{\lambda}^{(k)}\| \leq 10^{-9}$

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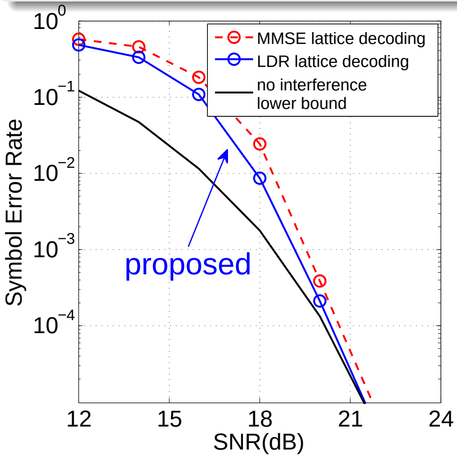

$M_C = N_C = 16, \;\; 16\text{-QAM}$

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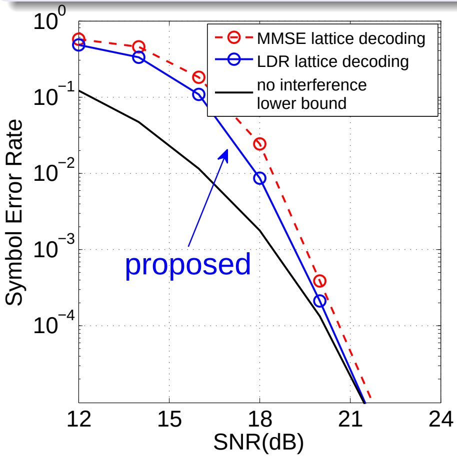
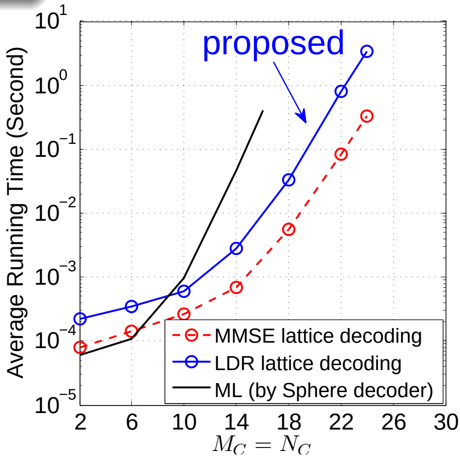
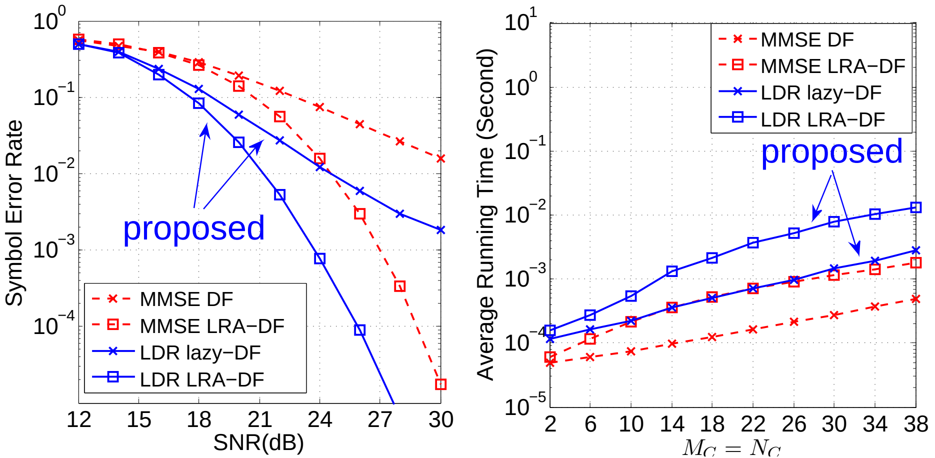


$M_C = N_C = 16$, 16-QAM



SNR=22dB, 16-QAM

LDR: Lattice Reduction aided DF and lazy DF



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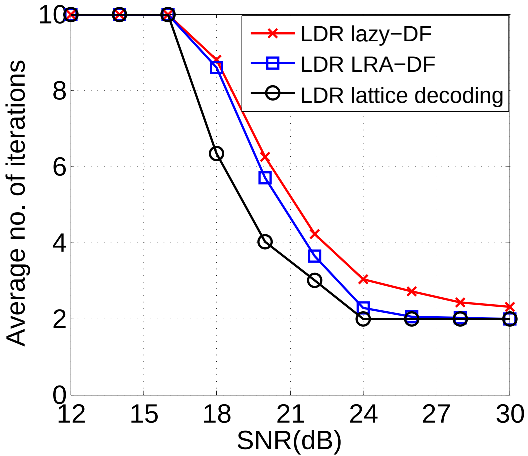
SNR=22dB, 16-QAM

No. of iterations



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Extensions

- Complexity reduction
  - A better starting point than MMSE
  - Successive radius update in lattice decoding
  - Successive lattice reduction

Thank you

Q & A

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