On Optimal Precoding in Wireless Multicast Systems

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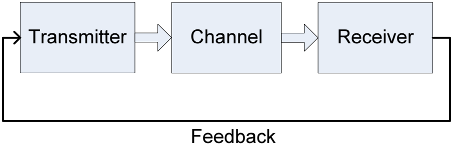
Outline

Motivation

- An adaptive wireless system with feedback



- Precoding improves transmission rate and reliability
  - Precoding codebook is shared by the transmitter and receiver
  - The user feeds back the index of the best precoder
- Precoding has been deployed in 4th generation wireless standards

Motivation, Cont'd

- Well investigated in point to point communications
  - Precoding codebook designs
  - Feedback delay and error
- Precoding for multiuser systems?
  - We consider multicast channel
  - Each user feeds back the index of its best precoder
- What is the optimal precoding selection scheme for the transmitter?

Assumptions

- Each user is treated equally
  - No admission control
  - Feedback treated with equal importance
- Consider typical LTE setups
  - The base station with two antennas
  - Each user equipped with two antennas
- Consider LTE precoding codebook

Receive Structure

- The received signal for user i over one time slot is given by

$$\mathbf{r}_i = \mathbf{H}_i \mathbf{P} \mathbf{x} + \mathbf{n}_i.$$

  where $\mathbf{P}$ is the precoding matrix.
- combine the received signals for all users as

$$\mathbf{R} = \mathbf{H} \mathbf{P} \mathbf{x} + \mathbf{n},$$

  where $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{r}_{N_u}]^T$, $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \cdots, \mathbf{H}_{N_u}]^T$ and $\mathbf{n} = [\mathbf{n}_1, \mathbf{n}_2, \cdots, \mathbf{n}_{N_u}]^T$.

MMSE Capacity

- For the link between the base station and the i^{th} user, the MMSE capacity is given as

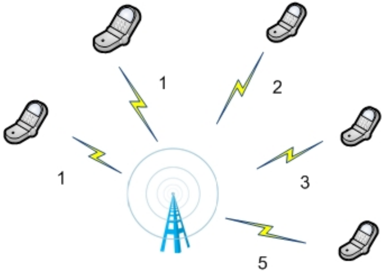
$$C_{0,\mathrm{MMSE}}(\mathbf{H}_i, \mathbf{P}) = \log\det\left(\mathbf{I} + \rho\mathbf{M}\right)$$

where $\mathbf{M}$ is a diagonal matrix with

$$\mathbf{M}_{jj} = \frac{1}{\left[(\mathbf{I} + \mathbf{P}^{\dagger}\mathbf{H}_i^{\dagger}\mathbf{R}_{\mathbf{n}_i}^{-1}\mathbf{H}_i\mathbf{P})^{-1}\right]_{jj}} - 1$$

and $\mathbf{R}_{\mathbf{n}_i}$ is the noise covariance matrix given by $\mathbf{R}_{\mathbf{n}_i} = \mathbb{E}[\mathbf{n}_i\mathbf{n}_i^{\dagger}]$.

Schemes



- Random selection
- Round Robin
- Majority votes
- We propose probabilistic algorithms
  - Geometry of codebooks
  - Channel dependent

Problem formulation

- We aim to minimize the average drop rate. We assume when outage occurs, the packet is dropped. So the packet drop rate for the i^{th} user is

$$\mathbf{1}_{\{C_0(\mathbf{H}_i,\mathbf{P}_k)<\lambda\}}$$

where λ is the required rate.

- The problem is formulated in the following:

$$k = \operatorname*{argmin}_k \frac{1}{N_u} \sum_{i=1}^{N_u} \mathbf{1}_{\{C_0(\mathbf{H}_i,\mathbf{P}_k)<\lambda\}} \big|_{k_1,k_2,\cdots,k_{N_u}}.$$

Since CSI is unknown at transmitter, it is hard to attack.

Problem reformulation

- Instead, we reformulate the following problem,

$$k = \operatorname*{argmin}_{k} \frac{1}{N_u} \sum_{i=1}^{N_u} \mathbb{E}_{k_i} \mathbf{1}_{\{C_0(\mathbf{H}_i, \mathbf{P}_k) < \lambda\}}$$

  where the expectation is over $\mathbf{H}_i$ and k_i is the index of the optimal precoder for the i^{th} user.

- Given a codebook and a stationary channel, it is feasible to evaluate

$$\mathbb{E}_{k_i} \mathbf{1}_{\{C_0(\mathbf{H}_i, \mathbf{P}_k) < \lambda\}}.$$

Algorithm procedure

- Step 1: Evaluate $\mathbb{E}_i \mathbf{1}_{\{C_0(\mathbf{H},\mathbf{P}_j)<\lambda\}}$: $\mathbf{A}$.

$$\mathbf{A}_{i,j} = \frac{1 - \mathbb{E}_i \mathbf{1}_{\{C_0(\mathbf{H},\mathbf{P}_j)<\lambda\}}}{1 - \mathbb{E}_i \mathbf{1}_{\{C_0(\mathbf{H},\mathbf{P}_i)<\lambda\}}}.$$

  So $\mathbf{A}$ is in the following form:

$$\mathbf{A} = \begin{bmatrix} 1 & \cdots & & \\ \cdots & 1 & \cdots & \\ & \cdots & 1 & \cdots \\ & & \cdots & 1 \end{bmatrix}$$

- Step 2: Summarize the feedback of the receivers as a vector $\mathbf{v}$, in which $\mathbf{v}_j$ denotes the number of occurrences of the preferred precoding matrix $\mathbf{P}_j$.

Algorithm procedure Cont'd

- Step 3: Select the optimal precoder:

$$k^* = \underset{k}{\text{argmax}}[\mathbf{Av}]_k.$$

Illustration:

$$\mathbf{Av} = \begin{bmatrix} 1 & \cdots & & \\ \cdots & 1 & \cdots & \\ & \cdots & 1 & \cdots \\ & & \cdots & 1 \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \\ \mathbf{v}_4 \end{bmatrix}$$

System Setup

- Channel model
  - stationary: $\mathbf{A}$ can be precalculated – offline.
  - non-stationary or unknown: adaptive learning of $\mathbf{A}$ – online.
- LTE precoding

Rank	Precoding Matrices
1	$\mathbf{Q}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{Q}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{Q}_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\mathbf{Q}_4 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{Q}_5 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ i \end{bmatrix}, \mathbf{Q}_6 = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -i \end{bmatrix}$
2	$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \frac{1}{2} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$

LTE precoding

- LTE precoding codebook

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- when SNR is low, rank 1 is optimal; Vice Versa, rank 2 is optimal.
- Construction of $\mathbf{A}$.

Parametrization of **A**

- We consider low SNR scenario in this presentation.
- Observations: three antipodal pairs
  - $\{\mathbf{Q}_1, \mathbf{Q}_2\}, \{\mathbf{Q}_3, \mathbf{Q}_4\}, \{\mathbf{Q}_5, \mathbf{Q}_6\}$.
- Parametrization of **A**:

$$\mathbf{A} = \begin{bmatrix} 1 & a & b & b & b & b \\ a & 1 & b & b & b & b \\ b & b & 1 & a & b & b \\ b & b & a & 1 & b & b \\ b & b & b & b & 1 & a \\ b & b & b & b & a & 1 \end{bmatrix}$$

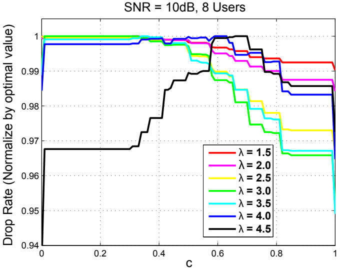
Parametrization of **A**, Cont'd

- Further Parametrization of **A**:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & c & c & c & c \\ 1 & 0 & c & c & c & c \\ c & c & 0 & 1 & c & c \\ c & c & 1 & 0 & c & c \\ c & c & c & c & 0 & 1 \\ c & c & c & c & 1 & 0 \end{bmatrix}$$

  where $c = \frac{1-b}{1-a}$.
- Assuming stationary Gaussian Channel model, the optimal c is a function of λ that can be precalculated.

Optimal c



- When λ is low, the optimal c is close to 0, namely $b \simeq 1$.
- When λ is high, the optimal c is close to 1, namely $a \simeq b$.

Adaptive learning of c

- When channel is non-stationary or the distribution is unknown, $\mathbf{A}$ needs to be learned.

Algorithm 1: Adaptive Algorithm

for $t \leftarrow 1$ **to** ∞ **do**
    $c' \leftarrow c_t + \sigma Z_t$
    $r' \leftarrow$ Average drop rate of strategy c'
    if $r' \leq r_t$ **then**
        $c_{t+1} \leftarrow c'$, $r_{t+1} \leftarrow r'$
    else
        With probability $e^{(r_t - r')/T}$
            $c_{t+1} \leftarrow c'$, $r_{t+1} \leftarrow r'$
        With probability $1 - e^{(r_t - r')/T}$
            $c_{t+1} \leftarrow c_t$
            $r_{t+1} \leftarrow (1-\alpha) r_t + \alpha r'$
    end
end

Relationship to R-factor

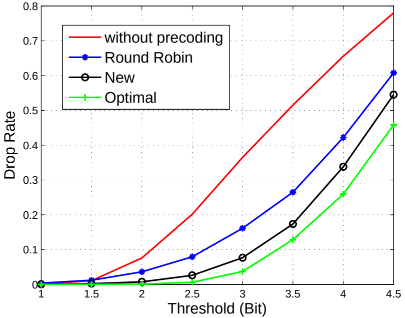
- For voice over IP (VoIP) traffic, the 'R-factor' γ is an indicator of voice quality.

$$\gamma = \gamma_\alpha - \alpha_1\delta - \alpha_2(\delta - \alpha_3)H - \beta_1 - \beta_2 \log(1 + 100\beta_3\psi)$$

where δ is the one-way end-to-end delay in milliseconds, ψ is the packet loss percentage and the other are codec dependent parameters.

- Packet drop rate has a direct impact on R-factor.

Improvement on Drop Rate



- Average packet drop rate for an 8-user multicast system with SNR $= 10$dB

Improvement on R-factor

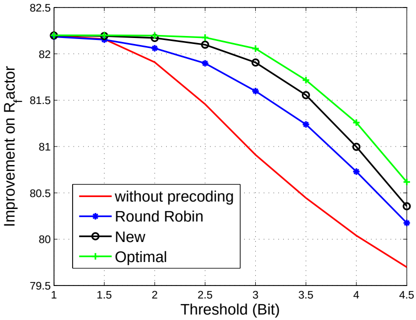
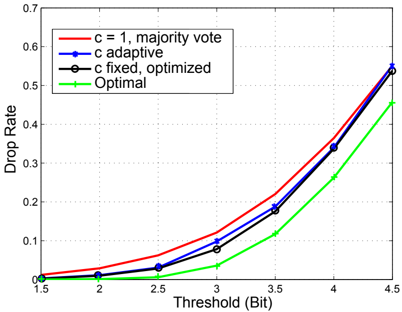


Figure: Improvement on the.

- R-factor for an 8-user multicast system with $\text{SNR} = 10\text{dB}$ assuming no delay.

Performance of different strategies



- When channel is stationary, adaptive algorithm approaches the offline algorithm.

Conclusions

- We have considered precoding selection in multi-cast channel.
- We have proposed a set of probabilistic algorithms to improve the packet drop rate
- When channel is stationary, access measurement can be precalculated. Vice versa, an adaptive learning algorithm is provided.

THANK YOU!