Convex Approximation Algorithms for Back-pressure Power Control of Wireless Multi-hop Networks

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Shortest path vs. dynamic back-pressure

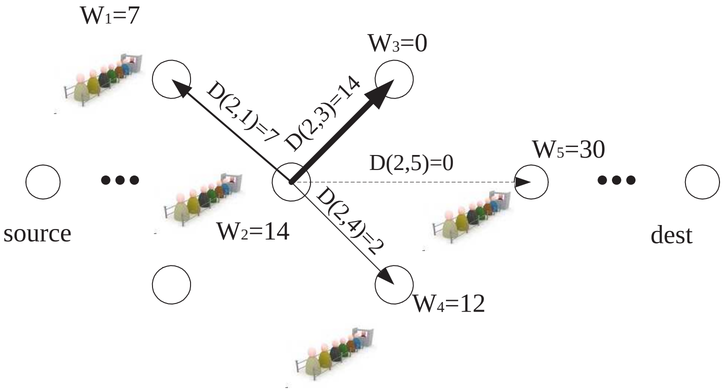
SP

- DP: BF, FW, ...
- Distributed ✓
- Must know arrival rate
- Quasi-static, very slow to adapt to
  - changing arrivals/load
  - availability/failure
  - fading/interference patterns
- Claim: Low delay (shortest path)
- ... but only at low system loads

BP [Tassiulas '92]

- One-hop differential backlog
- Distributed ✓ Lightweight ✓
- Auto-adapts ✓
- Highly dynamic, agile ✓
- Claim: maximal stable throughput (all paths)
- ... but delay can be large - U(load), $\emptyset \rightarrow$ rand walk!

Back-pressure routing



- Favors links with low back-pressure (hence name)
- Backtracking / looping possible!
- Local communication, trivial computation

Back-pressure routing

- Multiple destinations, commodities?
  - multiple queues per node
  - (max diff backlog) winner-takes-all per link
- Wireline: local communication, trivial computation
- Wireless?
- Broadcast medium: interference
- Link rates depend on transmission scheduling, power of other links
- Globalization - but also opportunity to shape-up playing field ...
- ... through appropriate scheduling, power control

Back-pressure power control

SINR

$$\gamma_\ell = \frac{G_{\ell\ell} p_\ell}{\sum_{k \in \mathcal{L}, k \neq \ell} G_{k\ell} p_k + V_\ell}$$

Link capacity

$$c_\ell = \log(1 + \gamma_\ell)$$

Diff backlog link $\ell = (i \rightarrow j)$ @ time t

$$D_\ell(t) := \max\{0, W_i(t) - W_j(t)\}$$

BPPC

$$\max_{\{p_\ell\}_{\ell \in \mathcal{L}}} \sum_{\ell \in \mathcal{L}} D_\ell(t) c_\ell$$

$$\text{s.t.} \quad 0 \leq \sum_{\ell : \mathrm{Tx}(\ell) = i} p_\ell \leq P_i, \forall i \in \mathcal{N}$$

$$p_\ell \leq P^{(\ell)}, \ell \in \mathcal{L}$$

Back-pressure power control

BPPC

$$\max_{\{p_\ell\}_{\ell\in\mathcal{L}}} \sum_{\ell\in\mathcal{L}} D_\ell(t)c_\ell$$

$$\text{s.t.} \quad 0 \le \sum_{\ell:\mathrm{Tx}(\ell)=i} p_\ell \le P_i, \forall i \in \mathcal{N}$$

$$p_\ell \le P^{(\ell)}, \ell \in \mathcal{L}$$

Link activation / scheduling:

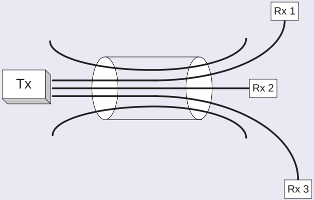
$$p_\ell \in \left\{0, P^{(\ell)}\right\}, \ell \in \mathcal{L}$$

[Tassiulas *et al*, '92 $\longrightarrow$]

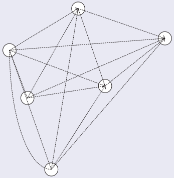
- Max stable throughput ✓
- Backbone behind modern NUM
- Core problem in wireless networking
- Countable control actions: random, adopt if > current
- Still throughput-opt! [Tass'98] - but D $\uparrow$
- Continuous opt vars?

Reminiscent of ...

DSL: sum-rate maximization



BPPC



Single-hop DSL

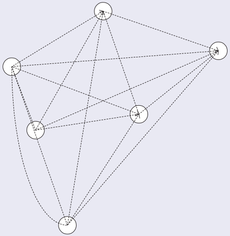
- Listen-while-talk ✓
- Dedicated (Tx,Rx)
- Free choice of $G_{k,\ell}$'s
- NP-hard [Luo, Zhang]

Multi-hop network

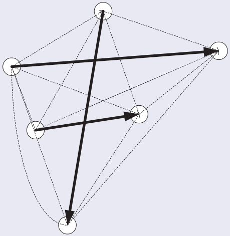
- No listen-while-talk X
- Shared Tx, Rx $\Rightarrow$
- Restricted $G_{k,\ell}$'s
- NP-hard?

Peel off

Generic backlogs



Choosing backlogs



DSL → Multi-hop network optimization

- Backlog reduction → BPPC contains DSL → also NP-hard
- Can reuse tools from DSL
- In particular, lower approximation algorithms:
  - High SINR → Geometric Programming
  - Successive approximation from below: SCALE [Papandriopoulos and Evans, 2006]
  - Uses

$$\alpha \log(z) + \beta \leq \log(1+z) \text{ for } \begin{cases} \alpha = \frac{z_o}{1+z_o} \\ \beta = \log(1+z_o) - \frac{z_o}{1+z_o}\log(z_o) \end{cases}$$

    tight at z_o; $\rightarrow \log(z) \leq \log(1+z)$ as $z_o \rightarrow \infty$
  - Start from high SINR, tighten bound at interim solution
  - Majorization (actually, minorization)

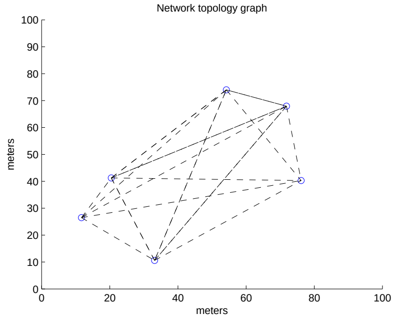
Key difference with DSL

- BPPC problem must be solved repeatedly for every slot
- Batch algorithms: prohibitive complexity
- Need adaptive, lightweight solutions (to the extent possible)
- Built custom interior point algorithms
- Normally, one would init using solution of previous slot; take refinement step
- Doesn't work ...
- Why?

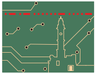
Proper warm-start

- No listen-while-talk, shared Tx/Rx
- Push-pull ‘wave’ propagation
- Solution from previous slot very different from one for present slot
- Even going back a few slots
- Quasi-periodic behavior emerges
- Idea: hold record of solutions for W previous slots. $W >$ upper bound on period
- W evaluations of present objective function (cheap!)
- Pick the best to warm-start present slot
- Needs few IP steps to converge

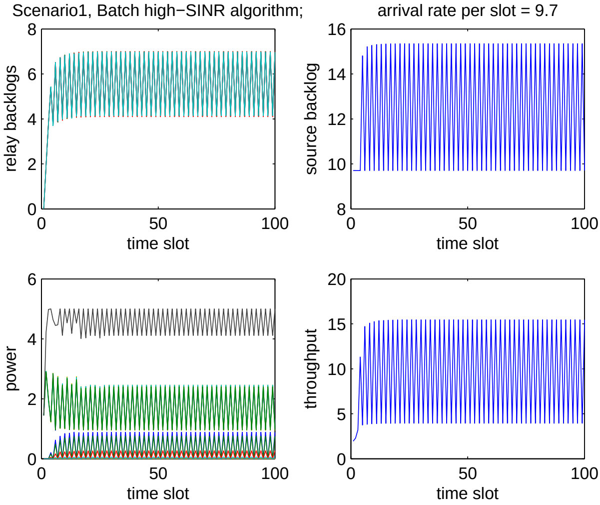
Simulation setup



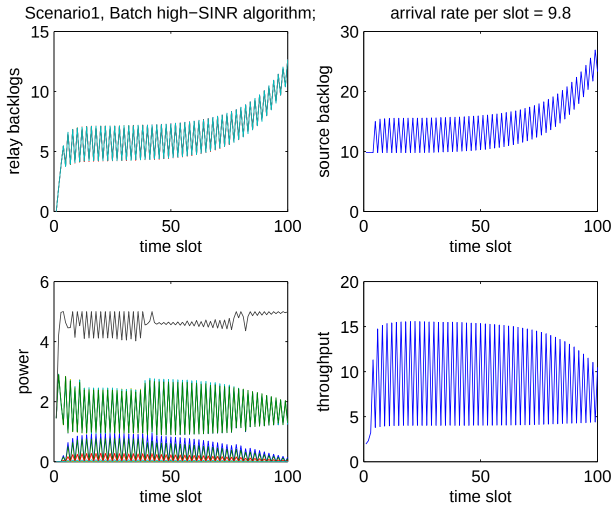
- $N = 6$ nodes, low-left = s, top-right = d, $L = 21$ links
- $G_{\ell,k} \sim 1/d^4$, $G = 128$, no-listen-while-talk $1/\text{eps}$
- $V_\ell = 10^{-12}$, $P^{(\ell)} = 5, \forall \ell$
- Deterministic (periodic) arrivals



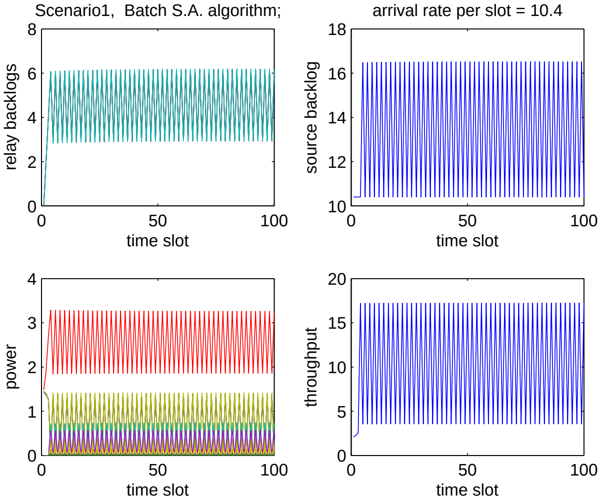
High SINR


Scenario1, Batch high−SINR algorithm;
arrival rate per slot = 9.7
relay backlogs
source backlog
power
throughput
time slot
0
50
100
8
6
4
2
0
16
14
12
10
8
6
4
2
0
20
15
10
5
0

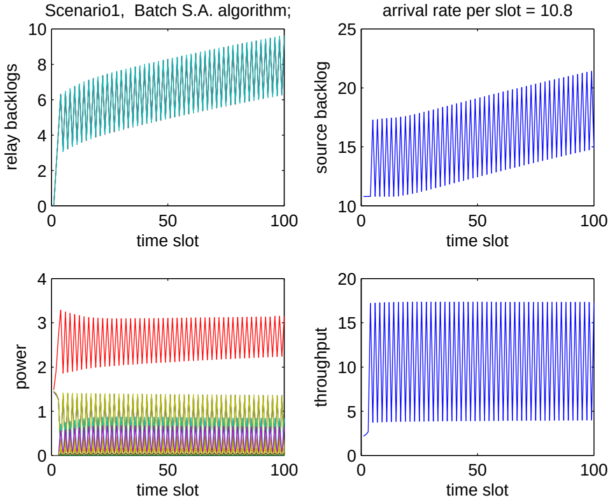
High SINR


Scenario1, Batch high−SINR algorithm;
arrival rate per slot = 9.8
relay backlogs
source backlog
power
throughput
time slot
0
5
10
15
20
30
4
6
2
50
100

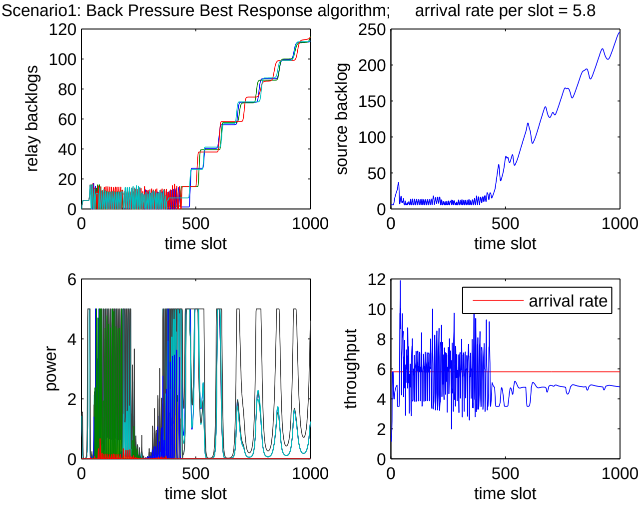
Successive Approximation



Successive Approximation



Best Response



Throughput comparison

Table: Attainable stable arrival rates in packets per slot.

Scenario	B/A high-SINR	B/A successive	Best Resp
Scenario 1	9.7	10.4	5.7
Scenario 2	2.4	7.5	2.1
Scenario 3	12.6	15.7	4.4

Ongoing & future work

Looking ahead

- Distributed BPPC
- Robustness (imperfect / outdated CSI)
- MIMO nodes - beamforming? precoding? spatial MUX?
- Other modalities - multicasting?
- All NP-hard, need effective approximation