OPTIMAL POWER ALLOCATION IN DISTRIBUTED MULTIPLE-RADAR CONFIGURATIONS

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OUTLINE

- Background
- Optimization metric
- Power allocation optimization: Minimize total transmit power
- Domain decomposition solutions
- Numerical analysis
- Concluding remarks

BACKGROUND

- Target localization estimation mean-square error (MSE) is lower bounded by the Cramér-Rao bound (CRB).
- In multiple radars architecture with widely separated antennas and non-coherent processing, the geometric spread provides a spatial related accuracy gain proportional to product of the number of transmitting and receiving radars [Godrich, Haimovich and Blum, IT 2010].
- In general, the localization MSE is a function of:
  - Number of transmit and receive radars
  - Geometric layout of the system with respect to the target
  - Signal effective bandwidth
  - Signal-to –noise ratio ⇨ transmitted power

PROBLEM DEFINITION

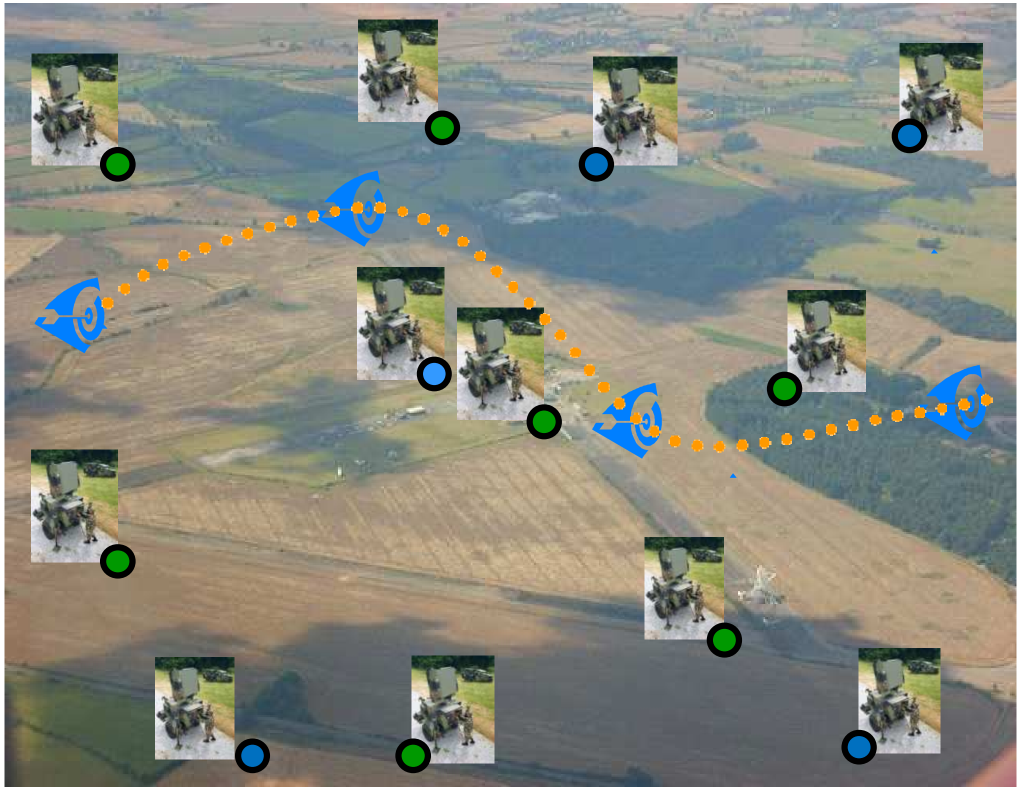
- There is an increasing number of applications that include mobile deployment of a large number of stations, such as ground surveillance radars (GSRs) for border patrol and security, observation and protection of remote areas, and tactical battlefield applications.
- In this scenario, the notion of resource-aware design is of critical importance. As each mission has its predetermined accuracy performance needs, adaptive power allocation algorithms may be applied for resource-aware operation.
- **Objective:** *Minimize the total transmitted power such that a predefined estimation MSE is attained, while keeping the transmitted power at each station within an acceptable range.*

SYSTEM MODEL

- Widely spread multiple radars system with M transmit radars and N receive radars.
- The target is modeled as an extended target, with a center of mass located at position (x,y).
- A set of M orthogonal signal is transmitted . The transmitted powers are:

$$\mathbf{p}_{tx} = \left[p_{1_{tx}}, p_{2_{tx}}, \dots, p_{M_{tx}}\right]^T$$

- Let $\tau_{m,n}$(x) denote the propagation time of a signal transmitted by radar m, reflected by to the scatterer, and received by radar n:



$$\tau_{m,n}(\mathrm{x}) = \left(\mathrm{R}_{m_{\mathrm{Rx}}} + \mathrm{R}_{n_{\mathrm{Rx}}}\right)/c$$

$$R_{m_{\mathrm{Tx}}} = \sqrt{\left(x_{m_{\mathrm{Tx}}} - x\right)^2 + \left(y_{m_{\mathrm{Tx}}} - y\right)^2}$$

$$R_{n_{\mathrm{Rx}}} = \sqrt{\left(x_{n_{\mathrm{Rx}}} - x\right)^2 + \left(y_{n_{\mathrm{Rx}}} - y\right)^2}$$

SIGNAL MODEL & OVERALL MSE BOUND

- The signal transmitted by radar m, reflected by the target, and received by radar n is: $r_{m,n}(t)=\sqrt{\alpha_{m,n}p_{m_{tx}}}h_{m,n}s_m(t-\tau_{m,n})+w_{m,n}(t)$

  where: $\alpha_{m,n}=\frac{k_{m,n}}{R_{m_{tx}}^2 R_{n_{rx}}^2}$ - variation is signal strength due to path loss.

  $h_{m,n}$ - complex coefficients that incorporate target reflectivity and phase offsets.

  $w_{m,n}(t)$ - complex white Gaussian process $\sim CN\left(0,\sigma_w^2\mathbf{I}\right)$.

- The overall variance of the x and y estimates is bounded by the trace of the Cramer-Rao bound (CRB) matrix:

$$\sigma_x^2+\sigma_y^2 \geq trace\left(\mathbf{C}_{x,y}\right)$$

  where the CRB matrix may be written as:

$$\mathbf{C}_{x,y}=\mathbf{J}^{-1}(\mathbf{u})=\left\{\sum_{m=1}^{M} p_{m_{tx}}\begin{bmatrix} g_{a_m} & g_{c_m} \\ g_{c_m} & g_{b_m}\end{bmatrix}\right\}^{-1}$$

THE CRB

- The terms in the CRB matrix are:

$$g_{a_m} = \kappa_m \sum_{n=1}^{N} \alpha_{m,n} \left|h_{m,n}\right|^2 \left(\frac{x_{m_{Tx}} - x}{R_{m_{Tx}}} + \frac{x_{n_{Rx}} - x}{R_{n_{Rx}}} \right)^2; \quad g_{b_m} = \kappa_m \sum_{n=1}^{N} \alpha_{m,n} \left|h_{m,n}\right|^2 \left(\frac{y_{m_{Tx}} - y}{R_{m_{Tx}}} + \frac{y_{n_{Rx}} - y}{R_{n_{Rx}}} \right)^2$$

$$g_{c_m} = \kappa_m \sum_{n=1}^{N} \alpha_{m,n} \left|h_{m,n}\right|^2 \left(\frac{x_{m_{Tx}} - x}{R_{m_{Tx}}} + \frac{x_{n_{Rx}} - x}{R_{n_{Rx}}} \right) \left(\frac{y_{m_{Tx}} - y}{R_{m_{Tx}}} + \frac{y_{n_{Rx}} - y}{R_{n_{Rx}}} \right); \quad \kappa_m = \frac{8\pi^2 \beta_m^2}{c^2 \sigma_w^2}$$

and $\mathbf{u} = [x, y, \mathbf{h}]$.

- The trace of the CRB matrix may be expressed as

$$trace(\mathbf{C}_{x,y}) = \frac{\mathbf{b}^T \mathbf{p}_{tx}}{\mathbf{p}_{tx}^T \mathbf{A} \mathbf{p}_{tx}}$$

$$\mathbf{b} = \mathbf{g}_a + \mathbf{g}_b; \quad \mathrm{A} = \mathbf{g}_a \mathbf{g}_b^T - \mathbf{g}_c \mathbf{g}_c^T$$

$$\mathbf{g}_a = \left[g_{a_1}, g_{a_2}, \ldots, g_{a_M}\right]^T; \mathbf{g}_b = \left[g_{b_1}, g_{b_2}, \ldots, g_{b_M}\right]^T; \mathbf{g}_c = \left[g_{c_1}, g_{c_2}, \ldots, g_{c_M}\right]^T$$

POWER ALLOCATION OPTIMIZATION
MINIMIZE TOTAL POWER

- Given a predetermined threshold for localization MSE denoted by $\eta_{\max}$, the total power is minimized such that the threshold MSE is achieved. The appropriate optimization problem is of the form :

$$\begin{array}{lll} \underset{\mathbf{p}_{tx}}{\text{minimize}} & \mathbf{1}^T \mathbf{p}_{tx} & \\ \text{s.t.} & tr\left(\mathbf{C}_{x,y}\left(\tilde{\mathbf{u}}\right)\right) \leq \eta_{\max} & \\ & p_{m_{tx}} \leq p_{m_{tx\max}}, & m = 1,\ldots, M \\ & p_{m_{tx}} \geq p_{m_{tx\min}}, & m = 1,\ldots, M \end{array}$$

where $\tilde{\mathbf{u}}$ is a vector of preliminary estimates of the target parameters, obtained in previous cycle. $p_{m_{tx\min}}$ and $p_{m_{tx\max}}$ are the minimal and maximal transmitted power for transmitter m, respectively.

POWER ALLOCATION OPTIMIZATION
KKT CONDITIONS

- The optimization problem is not convex. Using the expression for the CRB, the following relaxed convex optimization problem may be formulated:

$$\begin{aligned} \underset{\mathbf{p}_{tx}}{\text{minimize}} \quad & \mathbf{1}^T \mathbf{p}_{tx} \\ \text{s.t.} \quad & \eta_{\max} \mathbf{p}_{tx}^T \mathbf{A}_e \mathbf{p}_{tx} - \mathbf{b}_e^T \mathbf{p}_{tx} = 0 \\ & p_{m_{tx}} \leq p_{m_{tx\,\max}}, \quad m = 1,\ldots, M \\ & p_{m_{tx\,\min}} \geq p_{m_{tx}}, \quad m = 1,\ldots, M \end{aligned}$$

- The Lagrangian function for this optimization problem is:

$$\mathrm{L}\left(\mathbf{p}_{tx}, \lambda, \boldsymbol{\mu}, \boldsymbol{\upsilon}\right) = \mathbf{1}^T \mathbf{p}_{tx} + \lambda\left(\eta_{\max} \mathbf{p}_{tx}^T \mathbf{A}_e \mathbf{p}_{tx} - \mathbf{b}_e^T \mathbf{p}_{tx}\right) + \boldsymbol{\mu}^T\left(\mathbf{p}_{tx} - \mathbf{p}_{tx_{\max}}\right) + \boldsymbol{\upsilon}^T\left(\mathbf{p}_{tx_{\min}} - \mathbf{p}_{tx}\right)$$

POWER ALLOCATION OPTIMIZATION
KKT CONDITIONS (2)

- The appropriate *Karush-Kuhn-Tucker* (KKT) conditions are:

$$\nabla_{\mathrm{p}_{tx}} \mathrm{L} = 1 + \lambda^* \left(\eta_{\max} \left(\mathbf{A}_e + \mathbf{A}_e^T \right) \mathbf{p}_{tx}^* - \mathbf{b}_e \right) + \boldsymbol{\mu}^* - \boldsymbol{\upsilon}^*$$

$$\eta_{\max} \mathbf{p}_{tx}^{*T} \mathbf{A}_e \mathbf{p}_{tx}^* - \mathbf{b}_e^T \mathbf{p}_{tx}^* = 0$$

$$\mu_m^* \left(p_{m_{tx\,\max}} - p_{m_{tx}} \right) = 0 \qquad m = 1, \dots, M$$

$$\upsilon_m^* \left(p_{m_{tx}} - p_{m_{tx\,\min}} \right) = 0 \qquad m = 1, \dots, M$$

$$\mu_m^* \geq 0; \ \upsilon_m^* \geq 0; \ \lambda^* \geq 0; \qquad m = 1, \dots, M$$

- Choosing the inequality constraints to be inactive, $\mu_m = \upsilon_m = 0$, reduces the set of KKT conditions to:

$$\nabla_{\mathrm{p}_{tx}} \mathrm{L} = 1 + \lambda^* \left(\eta_{\max} \left(\mathbf{A}_e + \mathbf{A}_e^T \right) \mathbf{p}_{tx}^* - \mathbf{b}_e \right)$$

$$\eta_{\max} \mathbf{p}_{tx}^{*T} \mathbf{A}_e \mathbf{p}_{tx}^* - \mathbf{b}_e^T \mathbf{p}_{tx}^* = 0$$

$$\lambda^* \geq 0$$

POWER ALLOCATION OPTIMIZATION
OPTIMAL SOLUTION

- From the KKT conditions, the optimal power allocation vector, $\mathbf{p}_{tx}$, may be calculated as:

$$\mathbf{p}_{tx}^{*} = \frac{\mathbf{B}}{\eta_{\max}}\left(\mathbf{b}_e - \frac{1}{\lambda^*}\mathbf{1}\right)$$

$$\lambda_{1,2}^{*} = \pm\sqrt{\frac{\mathbf{1}^T\mathbf{B}^T\mathbf{A}_e\mathbf{B}\mathbf{1}}{\mathbf{b}_e^T\left(\mathbf{B}^T\mathbf{A}_e\mathbf{B} - \mathbf{B}\right)\mathbf{b}_e}}$$

$$\mathbf{B} = \left(\mathbf{A}_e + \mathbf{A}_e^T\right)$$

- The solution results in a Lagrangian leveling mechanism in the form of $\frac{1}{\lambda^*}\mathbf{1}$, which shifts the origin of the elements of vector $\mathbf{b}_e$ with respect to the level offset $\frac{1}{\lambda^*}$.

POWER ALLOCATION OPTIMIZATION
DOMAIN DECOMPOSITION

- For the decomposition, the transmit powers are regarded as M vertices, divided into two groups: one includes interior points and the other includes points located on the boundaries. We regroup vector $\mathbf{p}_{tx_K} = \left[\mathbf{p}_{tx1_K}^T, \mathbf{p}_{tx2_K}^T\right]^T$, where $\mathbf{p}_{tx2_K}$ includes either minimum or maximum power values and $\mathbf{p}_{tx1_K}$ is analytically calculated using the updated KKT conditions.

$$\mathbf{p}_{tx1_K}^* = \frac{1}{\eta_{\max}}\left(\left(\mathbf{B}_{11}\mathbf{b}_1 + \mathbf{B}_{21}\mathbf{b}_2\right) - \frac{1}{\lambda^*}\left(\mathbf{B}_{11}\mathbf{1} + \mathbf{B}_{21}\mathbf{1}\right)\right)$$

$$a_\lambda\left(\lambda^*\right)^2 + b_\lambda\lambda^* + c_\lambda = 0$$

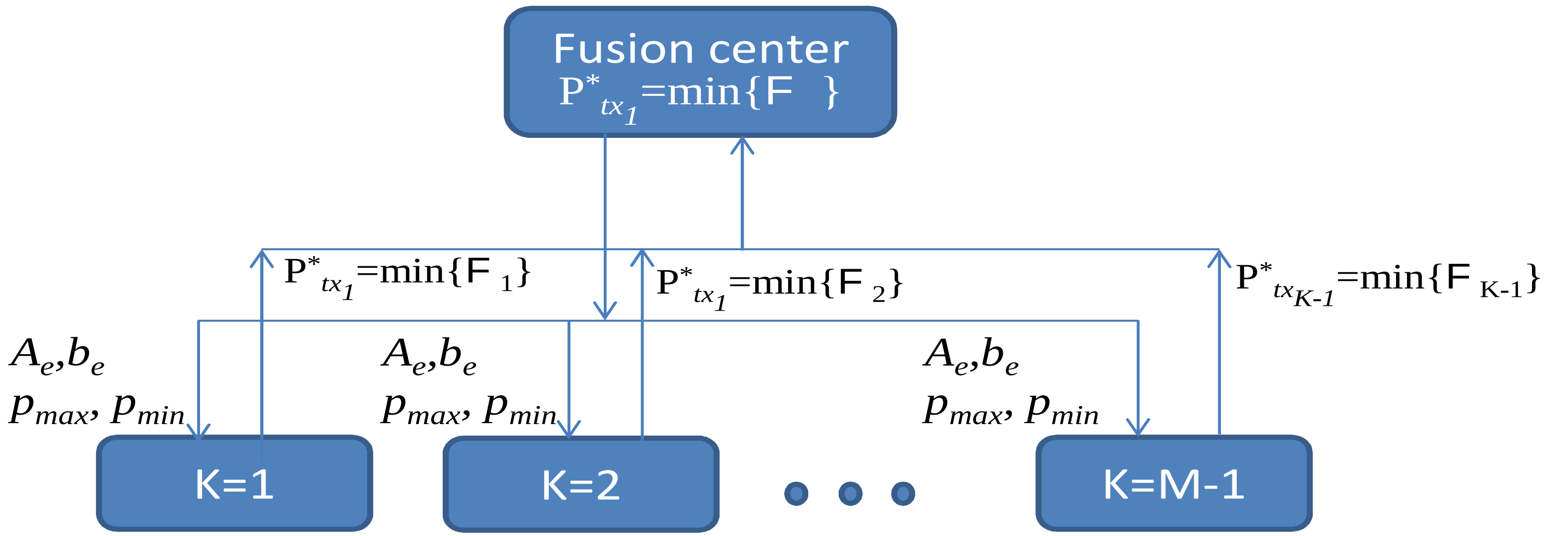
$$\mathbf{B} = \begin{bmatrix}\mathbf{B}_{11} & \mathbf{B}_{12}\\ \mathbf{B}_{21} & \mathbf{B}_{22}\end{bmatrix}; \mathbf{b}_e = \begin{bmatrix}\mathbf{b}_1\\ \mathbf{b}_2\end{bmatrix}$$

$$a_\lambda = f_1\left(\mathbf{B}, \mathbf{A}_e, \mathbf{b}_e, \eta_{\max}, \mathbf{p}_{tx2_K}\right); b_\lambda = f_2\left(\mathbf{B}, \mathbf{A}_e, \mathbf{b}_e, \eta_{\max}, \mathbf{p}_{tx2_K}\right)$$

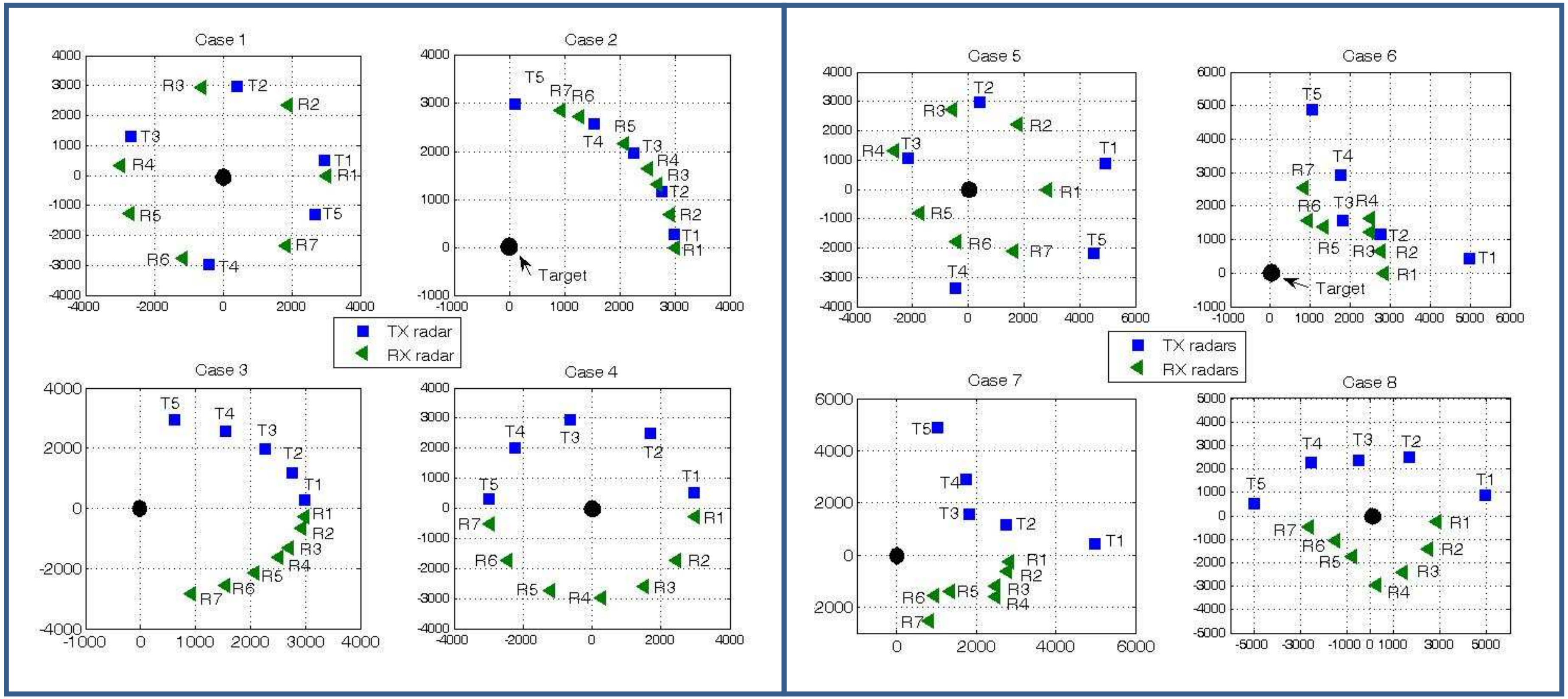
$$c_\lambda = f_3\left(\mathbf{B}, \mathbf{A}_e\right)$$

POWER ALLOCATION OPTIMIZATION
DOMAIN DECOMPOSITION (2)

- An original optimization problem is decomposed into M-1 sub-problems, supporting parallel processing where each sub-problem may be calculated at a different receiver or centrally, using parallel computing. A local processing center for the K-th sub-problem needs to get the values of vector **u**. Information of the local subset of optimal points F_K is transferred to the central processor.



NUMERICAL ANALYSIS
CONFIGURATIONS



NUMERICAL ANALYSIS
SYSTEM PARAMETERS

- Assume a 5x7 MIMO radar system.
- Matrix **H** values are:

$$\mathbf{H}_1 = \mathbf{1}_{7\times1}^T \mathbf{1}_{1\times5}$$

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 0.01 & 0.45 & 0.22 & 1 \\ 1 & 0.05 & 0.35 & 0.55 & 1 \\ 1 & 0.01 & 0.48 & 0.55 & 1 \\ 1 & 0.022 & 0.32 & 0.48 & 1 \\ 1 & 0.092 & 0.49 & 0.57 & 1 \\ 1 & 0.092 & 0.49 & 0.57 & 1 \\ 1 & 0.092 & 0.49 & 0.57 & 1 \end{bmatrix} \quad \mathbf{H}_3 == \begin{bmatrix} 0.1 & 1 & 1 & 1 & 0.75 \\ 0.05 & 1 & 1 & 1 & 0.4 \\ 0.01 & 1 & 1 & 1 & 0.45 \\ 0.12 & 1 & 1 & 1 & 0.55 \\ 0.09 & 1 & 1 & 1 & 0.3 \\ 0.2 & 1 & 1 & 1 & 0.2 \\ 0.19 & 1 & 1 & 1 & 0.25 \end{bmatrix}$$

- For uniform power allocation, the transmitted power is:

$$p_{tx_{uniform}}(total) = M \frac{\mathbf{b}^T \mathbf{1}}{\eta_{\max} \mathbf{1}^T \mathbf{A1}}$$

NUMERICAL ANALYSIS
EXAMPLE 1

- For a given MSE performance threshold, η_{max}=10 m^2 and $\mathbf{H}_2$, the following power allocation vector is generated:

	Case 1	Case 2	Case 3	Case 4
P_{total} – Uniform	162	189	332	141
P_{total}	90 (56%)	89 (47%)	151 (46%)	145 (47%)
$\mathbf{P}_{tx}$	$\begin{bmatrix}37\\1\\1\\1\\50\end{bmatrix}$	$\begin{bmatrix}36\\1\\1\\1\\50\end{bmatrix}$	$\begin{bmatrix}68\\1\\1\\1\\80\end{bmatrix}$	$\begin{bmatrix}80\\1\\1\\1\\62\end{bmatrix}$

NUMERICAL ANALYSIS
EXAMPLE 2

- For a given MSE performance threshold, η_{max}=10 m^2 and $\mathbf{H}_3$, the following power allocation vector is generated:

	Case 1	**Case 2**	**Case 3**	**Case 4**
P_{total} – Uniform	116	281	442	369
P_{total}	77 (66%)	161 (75%)	279 (63%)	208 (56%)
$\mathbf{P}_{tx}$	$\begin{bmatrix} 1 \\ 34 \\ 40 \\ 1 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 80 \\ 1 \\ 78 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 150 \\ 1 \\ 126 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 100 \\ 99 \\ 7 \\ 1 \end{bmatrix}$

NUMERICAL ANALYSIS
EXAMPLE 3

- For a given MSE performance threshold, η_{max}=10 m^2 and $\mathbf{H}_1$, the following power allocation vector is generated:

	Case 5	Case 6	Case 7	Case 8
P_{total} – Uniform	60	132	224	180
P_{total}	39 (65%)	118 (90%)	195 (87%)	103 (57%)
$\mathbf{P}_{tx}$	$\begin{bmatrix}1\\4\\22\\11\\1\end{bmatrix}$	$\begin{bmatrix}1\\10\\105\\1\\1\end{bmatrix}$	$\begin{bmatrix}1\\87\\105\\1\\1\end{bmatrix}$	$\begin{bmatrix}1\\1\\99\\1\\1\end{bmatrix}$

CONCLUDING REMARKS

- Resource-aware operation of distributed multiple radar systems is supported through a power allocation scheme that minimizes the total radiating power for a predetermined target localization MSE threshold.
- The power allocation nonconvex optimization problem has been solved through domain decomposition methods. The resulting parallel set of optimization problems supports distributed processing.
- The power allocation expressions reveal a Lagrangian leveling mechanism that weights the overall system conditions for a given transmitter and allocates power appropriately.
- Uniform power allocation is not necessarily the best choice and significant power savings can be obtained through the proposed strategy.

Q & A

Optimal Power Allocation in Distributed Multiple-Radar Configurations

PHYSICAL INTERPRETATION

- Time delay estimation error is: $\sigma^2_{\tau_{m,n}} \propto \dfrac{\sigma_w^2}{\left|h_{m,n}\right|^2 \alpha_{m,n} p_{m_{tx}} \beta^2}$
- This translates to:

$$MSE\left(\hat{R}_{m_{Tx}} + \hat{R}_{n_{Rx}}\right) \propto \frac{c^2\sigma_w^2}{\left|h_{m,n}\right|^2 \alpha_{m,n} p_{m_{tx}} \beta^2}$$