



# INFORMED SECURE WATERMARKING USING OPTIMAL TRANSPORT

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# Outline

- 1 Introduction
- 2 Requirements for secure embedding
- 3 Performance analysis
- 4 Conclusions and perspectives



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# Security in Watermarking

## Adversary

- Limited or unlimited power

## Materials

- Watermark detector/decoder
- Contents watermarked with the same key/messages
- Knowledge about the embedding and decoding algorithm (Kerckhoffs' principle)

## Objectives

- Estimate the secret key
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# Security framework

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- Gaussian host
- i.i.d. host
- Same secret key
- Watermarked contents only: WOA

## Goals

- Design a secure scheme
- Minimise the embedding distortion
- Maximise the robustness



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# How to maximise robustness?

- Use informed coding and side information from the Host (Costa's idea)
- Generate different coding regions for the same message
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# 1 :Distribution Splitting (I/II)

## Stego-security a.k.a. Perfect Secrecy

- $p(x) = p(y|k)$

## Binary Embedding

- $p_{Y|K} = (p_{Y|K,m=0} + p_{Y|K,m=1})/2$

## Partitioning function $g(\cdot)$

- $p_{Y|K,m=0}(x) = 2g(x)p_X(x)$
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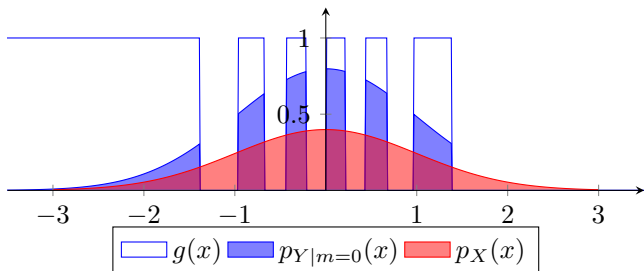
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# 1: Distribution Splitting (II/II)





## 2: Distribution Matching



### Requirement for Distribution Matching

- Find a mapping  $y = T(x)$  such that  $x \sim p_X$  and  $y \sim p_{Y|m=0,1}$



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## 3: Distortion Minimisation

- Minimise the average  $L^2$  distortion

Solution for distortion matching and distortion minimisation:

- 1 Optimal Transportation
- 2  $T(x) = F_{Y|d}^{-1} \circ F_X(x)$ ,
- 3  $\sigma_w^2 = \int_0^1 (F_Y^{-1}(x|d) - F_X^{-1}(x))^2 dx$



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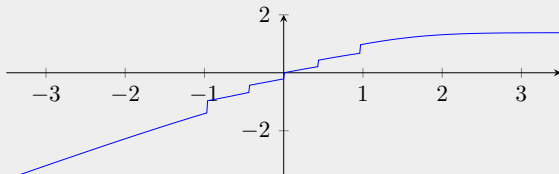
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# Example

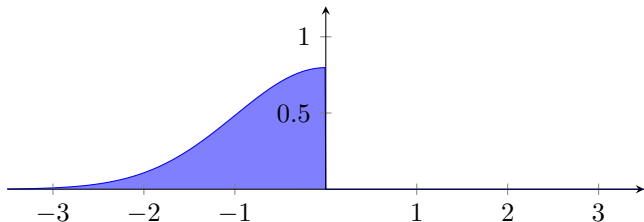


## Example of $T(x)$



## Different partitioning functions I/IV

Transport Natural Watermarking (TWN) [1] (non informed):

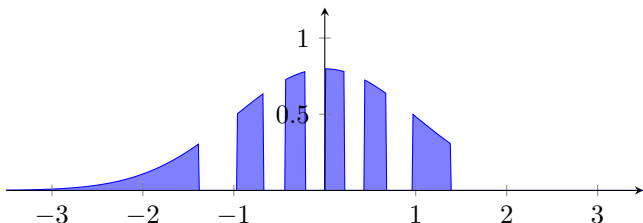


[1] B. Mathon, P. Bas, F. Cayre, and B. Macq, "**Optimization of Natural Watermarking Using Transportation Theory**" in MM&Sec'09 : Proceedings of the 11th ACM workshop on Multimedia and security, New York, NY, USA, 2009, pp. 33–38, ACM.



## Different partitioning functions II/IV

$p$  Natural Watermarking ( $p$ -NW): each coding region has the same probability  $p$ :

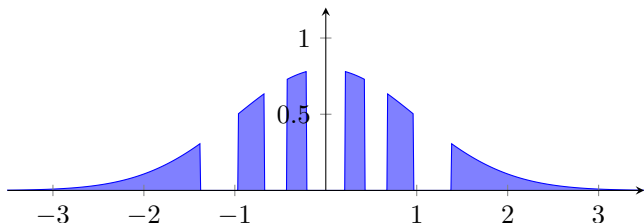






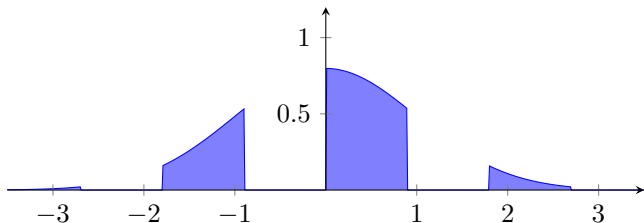
# Different partitioning functions III/IV

$\bar{\rho}$  Natural Watermarking ( $\bar{\rho}$ -NW):  $\rho$  Natural Watermarking symmetrised on 0:



## Different partitioning functions IV/IV

$\Delta$  Natural Watermarking ( $\Delta$ -NW): each coding region has the same width:



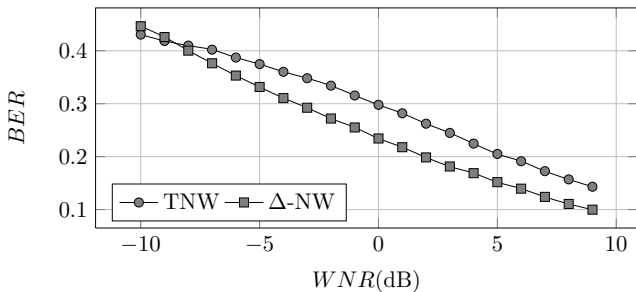


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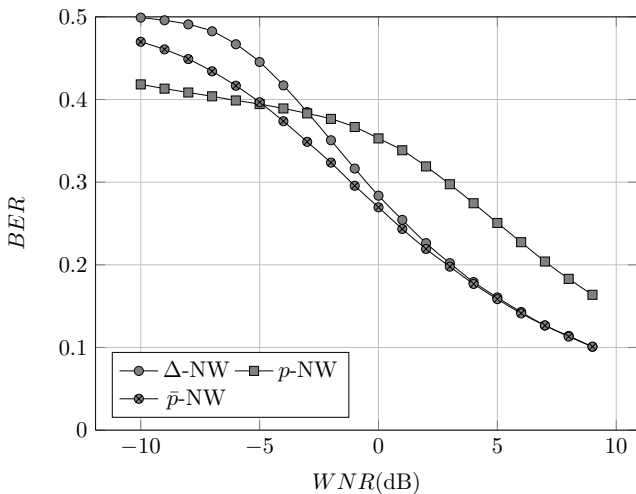


# Non-informed Vs informed coding



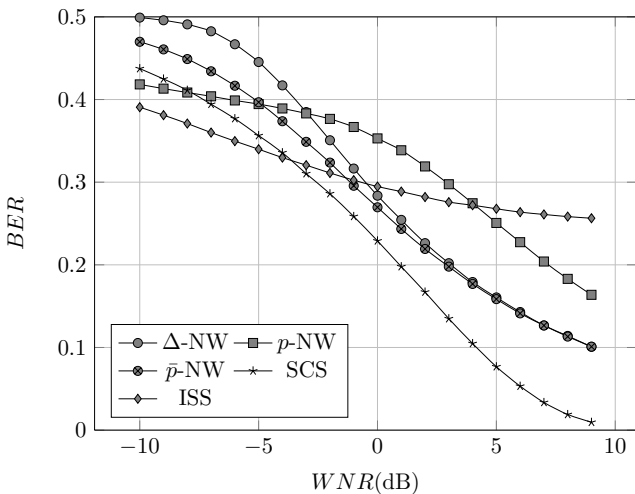


$$WCR = -5dB$$



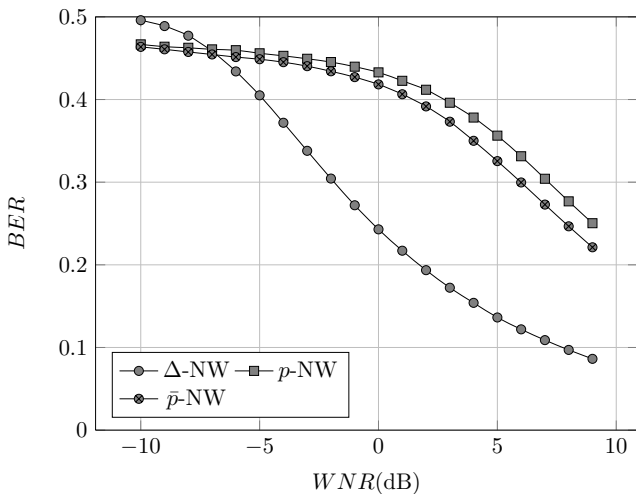


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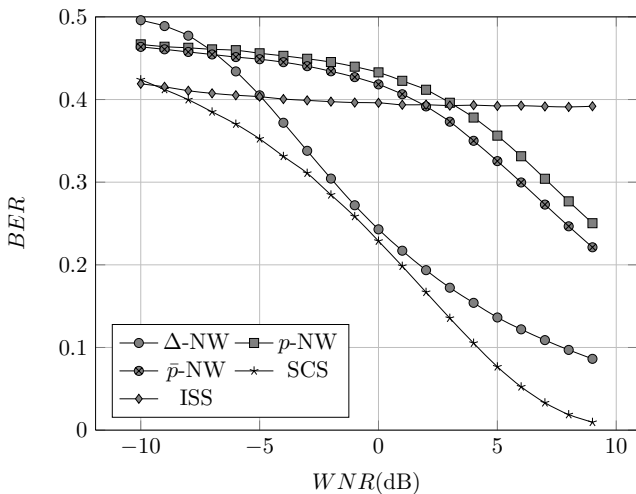




$$WCR = -11dB$$





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- Optimal transport adapted in 1-D
- Gain of using Informed Coding
- The best partitioning depends of the  $WCR$  and the  $WNR$

## Perspectives

- Links with secure adaptations of SCS (IH11)
- What is the secure capacity?
- Can we find secure error correcting codes to increase the quality of service?



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