INSTANTANEOUS PHASE TRACKING OF OSCILLATORY SIGNALS USING EMD AND RAO-BLACKWELLISED PARTICLE FILTERING

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Introduction

- Empirical mode decomposition
- Problem formulation using PF
- Problem formulation using RBPF
- New phase tracking system
- Experimental results
- Conclusion

Empirical mode decomposition

- It is adaptive and data-driven method for analysing nonlinear and nonstationary signals
- It Decomposes a mixture signal to its oscillatory waveforms called intrinsic mode functions (IMFs)
- IMFs are noisy especially the first few IMFs
- Sum of the resulting IMFs is equal to the original signal

$$\mathbf{X}_t = \sum_{i=1}^{M} imf_t(i)$$

Problem formulation using PF

$$imf_t(i) + J\mathrm{H}[imf_t(i)] = a_t(i)e^{J\theta_t(i)} \qquad i = 1,\ldots,M$$

$$\mathbf{X}_t = \sum_{i=1}^{M} imf_t(i) = \mathbf{Real}\,(\sum_{i=1}^{M} a_t(i)e^{J\theta_t(i)})$$

$$\mathbf{R}_t = [\theta_t(1) \quad a_t(1) \quad \ldots. \quad \theta_t(M) \quad a_t(M)]$$

$$\mathbf{R}_t = f(\mathbf{R}_{t-1}) + \mathrm{w}_{t-1}$$

$$\mathbf{X}_t = G(\mathbf{R}_t) + \mathrm{v}_t$$

$$G(\mathbf{R}_t) = \mathbf{Real}\,(\sum_{i=1}^{M} a_t(i)e^{J\theta_t(i)})$$

Problem formulation using RBPF

$$\mathbf{R}_t = [\mathbf{R}_t^1 \quad \mathbf{R}_t^2]$$

$$\mathbf{R}_t^1 = [\theta_t(1) \quad \ldots \quad \theta_t(M)]$$

$$\mathbf{R}_t^2 = [a_t(1) \quad \ldots \quad a_t(M)]$$

NOT KNOWN

$$\mathbf{R}_t = f(\mathbf{R}_{t-1}) + \mathrm{w}_{t-1}$$

$$\mathbf{X}_t = G(\mathbf{R}_t) + \mathrm{v}_t$$

$$G(\mathbf{R}_t) = \mathbf{R}_t^2 G'(\mathbf{R}_t^1)$$

$$= [a_t(1) \quad \ldots \quad a_t(M)] \begin{bmatrix} \mathrm{Real}(e^{J\theta_t(1)}) \\ \ldots \\ \mathrm{Real}(e^{J\theta_t(M)}) \end{bmatrix}$$

New phase tracking system

- Estimating instantaneous frequency using derivation of phase:

$$w_t(i) = \frac{d\theta_t(i)}{dt} \qquad for \qquad i = 1, \ldots, M$$

- The frequency trances of the noisy IMF are not smooth
- Track the instantaneous phases while smoothing the frequency traces

- Amplitude and phase tracking from a noisy IMF using the new RBPF-based system:

$$\mathrm{R}_t(i) = [\theta_t(i) \quad a_t(i)], \qquad \mathrm{R}_t^1(i) = \theta_t(i), \qquad \mathrm{R}_t^2(i) = a_t(i)$$

$$\mathrm{R}_t(i) = \tilde{f}(\mathrm{R}_{t-1}(i)) + \mathrm{w}_{t-1}$$

$$imf_t(i) = \tilde{G}(\mathrm{R}_t(i)) + \mathrm{v}_t$$

$$\tilde{G}(\mathrm{R}_t(i)) = a_t(i)\mathbf{Real}(e^{J\theta_t(i)})$$

select i^{th} IMF as $imf(i)$, $n = 1, ..., N$, *N is the number of particles.*
set $t = 0$ *and generate random numbers* $\theta_0^{(n)}, freq_0^{(n)}$
according to random uniform distribution considering the HT of the IMF.
Initialize $\sigma, \sigma_1, min_f, max_f, Ts$ *(Sampling period),* τ
for $\{t = 1$ *to* $t_{max}\}$ $\{t_{max}$ *is the number of all time samples*$\}$
- ***generate*** *random numbers* $\mathbf{w}_t^{\rho(n)1}, \mathbf{w}_t^{\rho(n)2}$ *and* $\mathbf{z}_t^{\rho(n)}$
- ***set*** $\hat{\theta}_t^{(n)} = \theta_{t-1}^{(n)} + \mathbf{w}_t^{\rho(n)1}$
- ***set*** $\breve{\theta}_t^{(n)} = -\theta_{t-1}^{(n)} + \mathbf{w}_t^{\rho(n)2}$
- ***Calculate*** $freq1 = diff(unwrap\,[\theta_{t-1}^{(n)} \quad \hat{\theta}])/Ts/(2 \times \pi)$
- ***Calculate*** $freq2 = diff(unwrap\,[\theta_{t-1}^{(n)} \quad \breve{\theta}])/Ts/(2 \times \pi)$
- ***Estimate frequency*** $freq = freq_{t-1}^{(n)} + \mathbf{z}_t^{\rho(n)}$
- ***Set*** $\hat{\theta}_t^{(n)}$ *as the phase, estimate amplitude* $\mathbf{a}_1$ *by Kalman filtering*
- ***Set*** $\mathbf{F}_1 = \mathbf{a}_1 \times Real(e^{J\hat{\theta}_t^{(n)}})$,
  $W_{s1} = exp\,(-(imf_t(i) - \mathbf{F}_1) \times (imf_t(i) - \mathbf{F}_1)/(2 \times \sigma_1^2))$
- ***Set*** $\breve{\theta}_t^{(n)}$ *as the phase, estimate amplitude* $\mathbf{a}_2$ *by Kalman filtering*
- ***Set*** $\mathbf{F}_2 = \mathbf{a}_2 \times Real(e^{J\breve{\theta}_t^{(n)}})$,
  $W_{s2} = exp\,(-(imf_t(i) - \mathbf{F}_2) \times (imf_t(i) - \mathbf{F}_2)/(2 \times \sigma_1^2))$
- ***if*** $(|\hat{\theta}_t^{(n)}| < \tau$ *and* $|\theta_{t-1}^{(n)}| < \tau)$ *or* $(|\breve{\theta}_t^{(n)}| < \tau$ *and* $|\theta_{t-1}^{(n)}| < \tau)$
  $weight1 = W_{s1}, weight2 = W_{s2}$
  $freq1 = freq,\ freq2 = freq, W_{s1} = 1, W_{s2} = 1$
- ***else***
  -$weight1 = exp\,(-(freq - freq1) \times (freq - freq1)/(2 \times \sigma^2))$
  -$weight2 = exp\,(-(freq - freq2) \times (freq - freq2)/(2 \times \sigma^2))$
- ***end if***
- ***if*** $weight1 > weight2$
  $\theta_t^{(n)} = \hat{\theta}_t^{(n)}, freq_t^{(n)} = freq1, w_t^{(n)} = w_{t-1}^{(n)} \times W_{s1} \times weight1$
- ***else***
  $\theta_t^{(n)} = \breve{\theta}_t^{(n)}, freq_t^{(n)} = freq2, w_t^{(n)} = w_{t-1}^{(n)} \times W_{s2} \times weight2$
- ***end if***
- ***if*** $(\theta_t^{(n)} > \pi)$ *or* $(\theta_t^{(n)} < -\pi)$ $\quad w_t^{(n)} = 0$
- ***if*** $(freq_t^{(n)} > max_f)$ *or* $(freq_t^{(n)} < min_f)$ $\quad w_t^{(n)} = 0$
- ***Normalize particle weights*** $w_t^{(n)} = w_t^{(n)} / \sum_{n=1}^{N}(w_t^{(n)})$
- ***Resample***

- end for

Experimental results

- **Simulated data**

  Sum of four amplitude and frequency modulated sine waves plus Gassian white noise (GWN)

  SNR is calculated as:

$$\text{SNR} = 10\log\left(\frac{P_{signal}}{P_{noise}}\right)$$

Two sets of simulated data SNR = 3.0445dB and SNR = 7.2167dB

MSE is defined as:

$$\text{MSE} = 1/T\sum_{t=1}^{T}(\theta_{estimate}(t) - \theta_{actual}(t))^2$$

Table 1. MSE of the phase in two SNR levels

	SNR=3.0445dB	SNR=7.2167dB
Proposed method	3.7634	2.6952
HT of the noisy IMF	5.7522	3.5871

Estimation of instantaneous phase

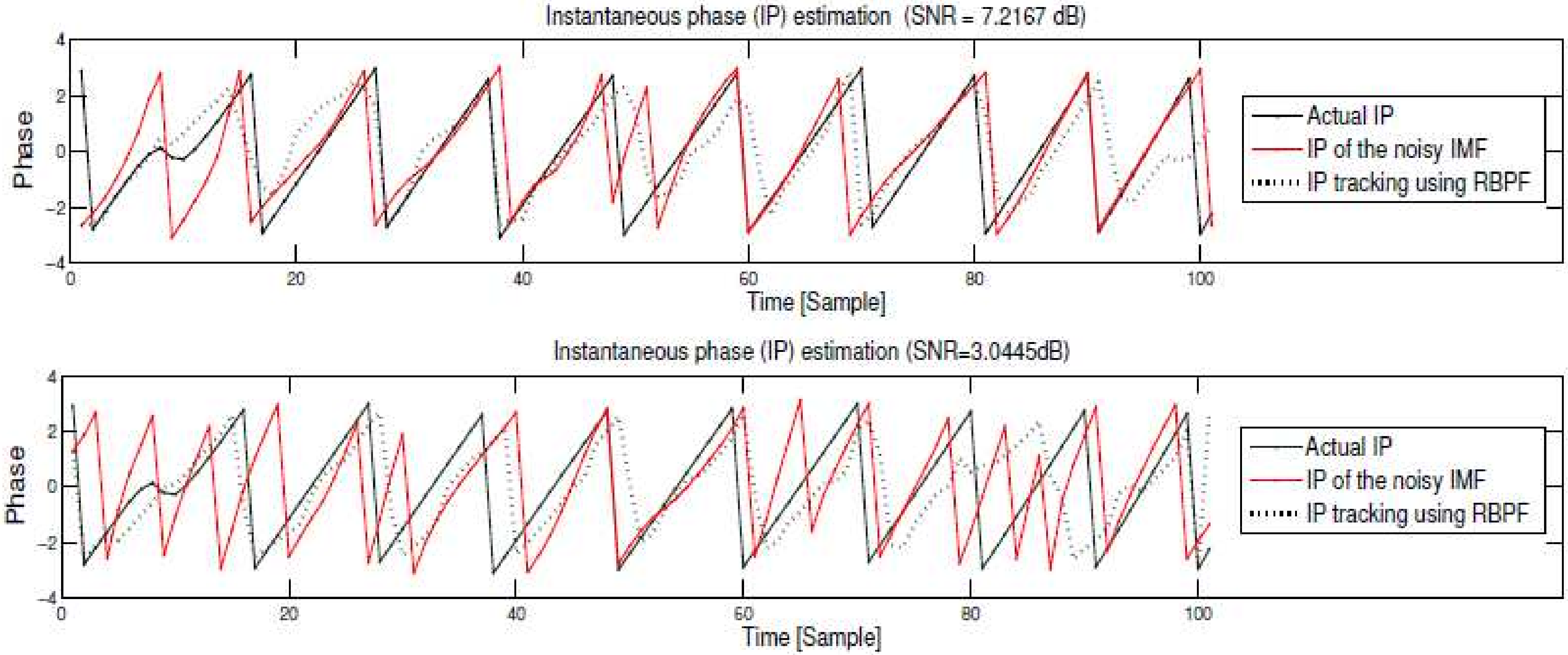

Fig. 1. IP estimation using the proposed and HT methods in two SNR levels

Estimation of instantaneous amplitude

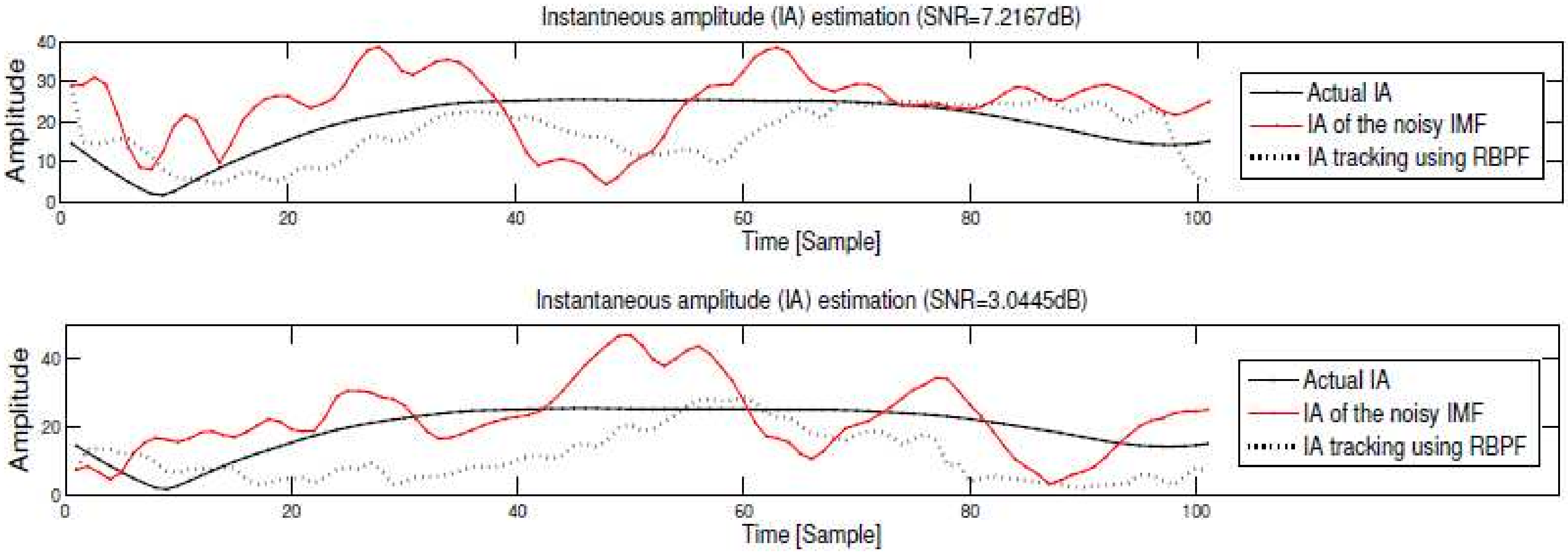

Fig. 2. IA estimation using the proposed and HT methods in two SNR levels

Estimation of instantaneous frequency

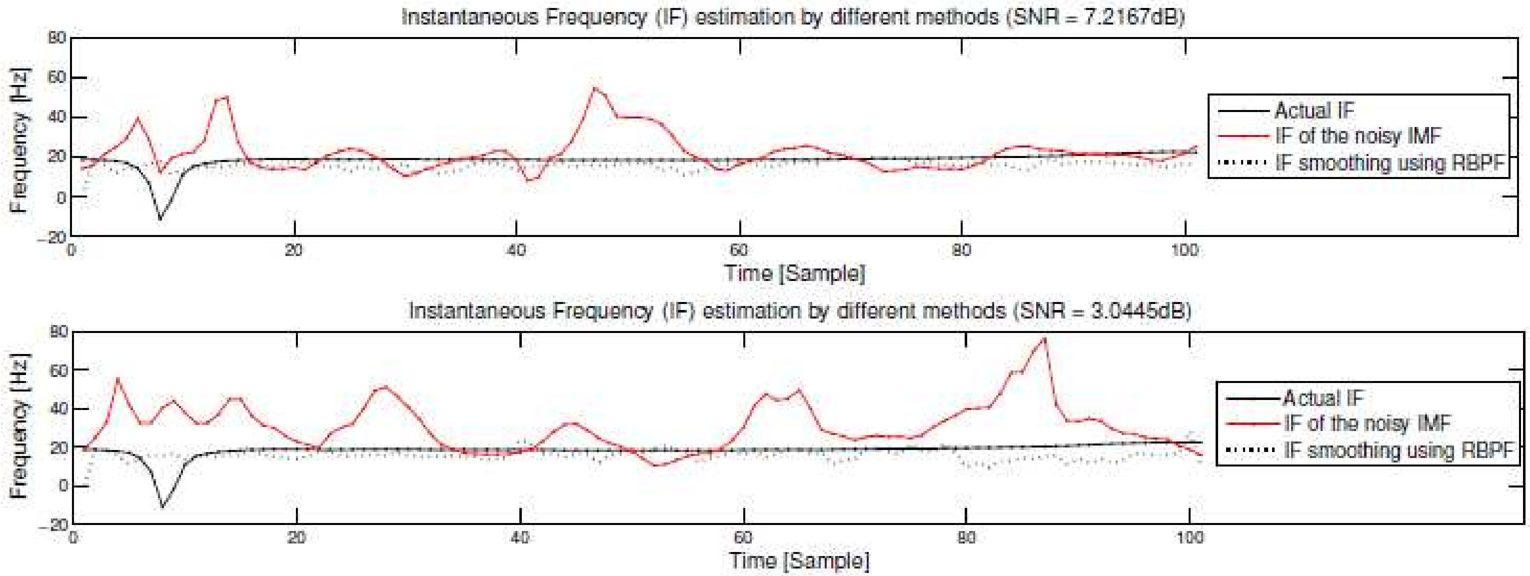

Fig. 3. IF estimation using the proposed and HT methods in two SNR levels

- **Real data**

Estimation of instantaneous phase and instantaneous amplitude

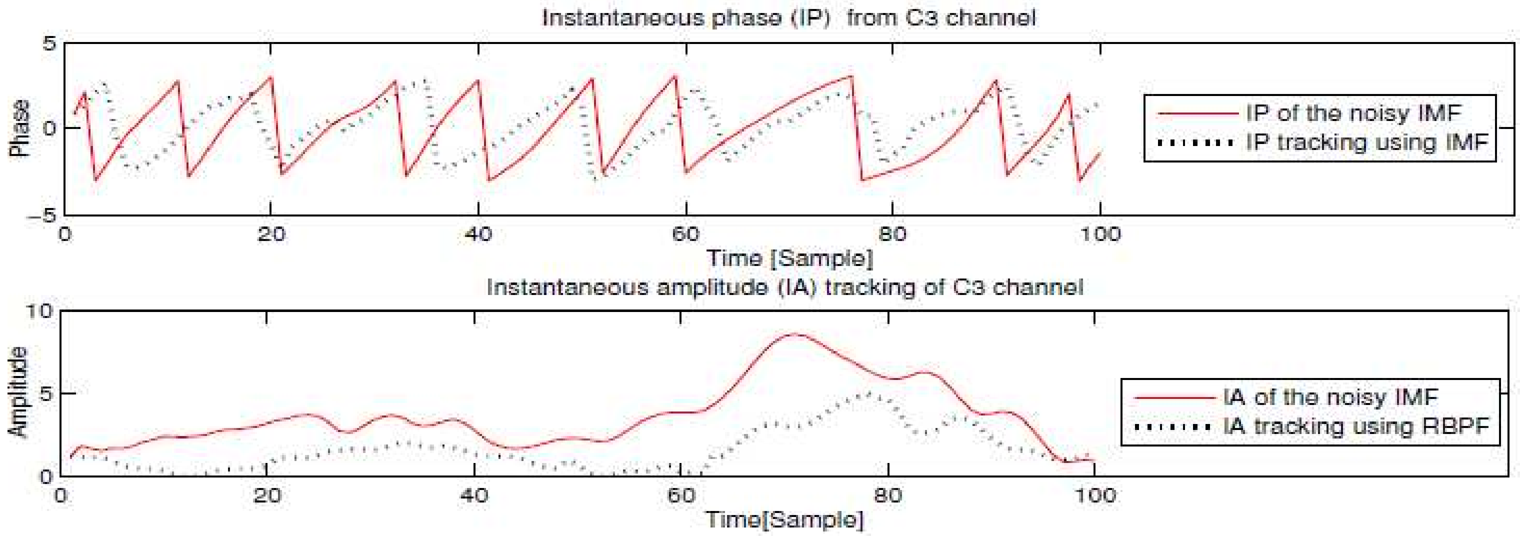

Fig. 4. IP and IA estimation of C3 channel using the proposed method

Estimation of instantaneous frequency

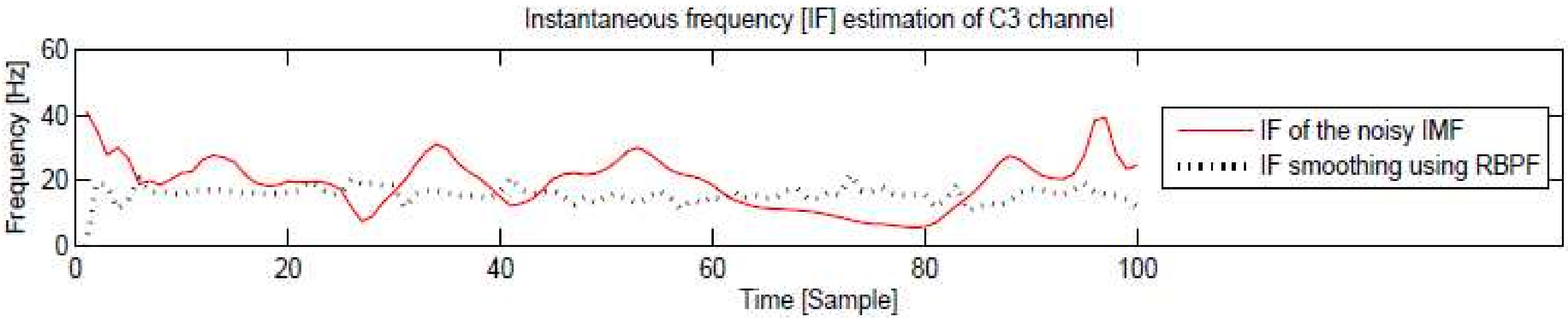

Fig. 5. IF estimation of C3 channel using the proposed method

Conclusion

- New phase tracking system based on EMD and RBPF
- Smoothing frequency traces and adding constraints to the RBPF formulation
- Restoring/denoising the IMFs
- Solving the mode mixing problem of EMD
- Application to speech enhancement

THANK YOU!