Compression of QRS complexes using Hermite expansion

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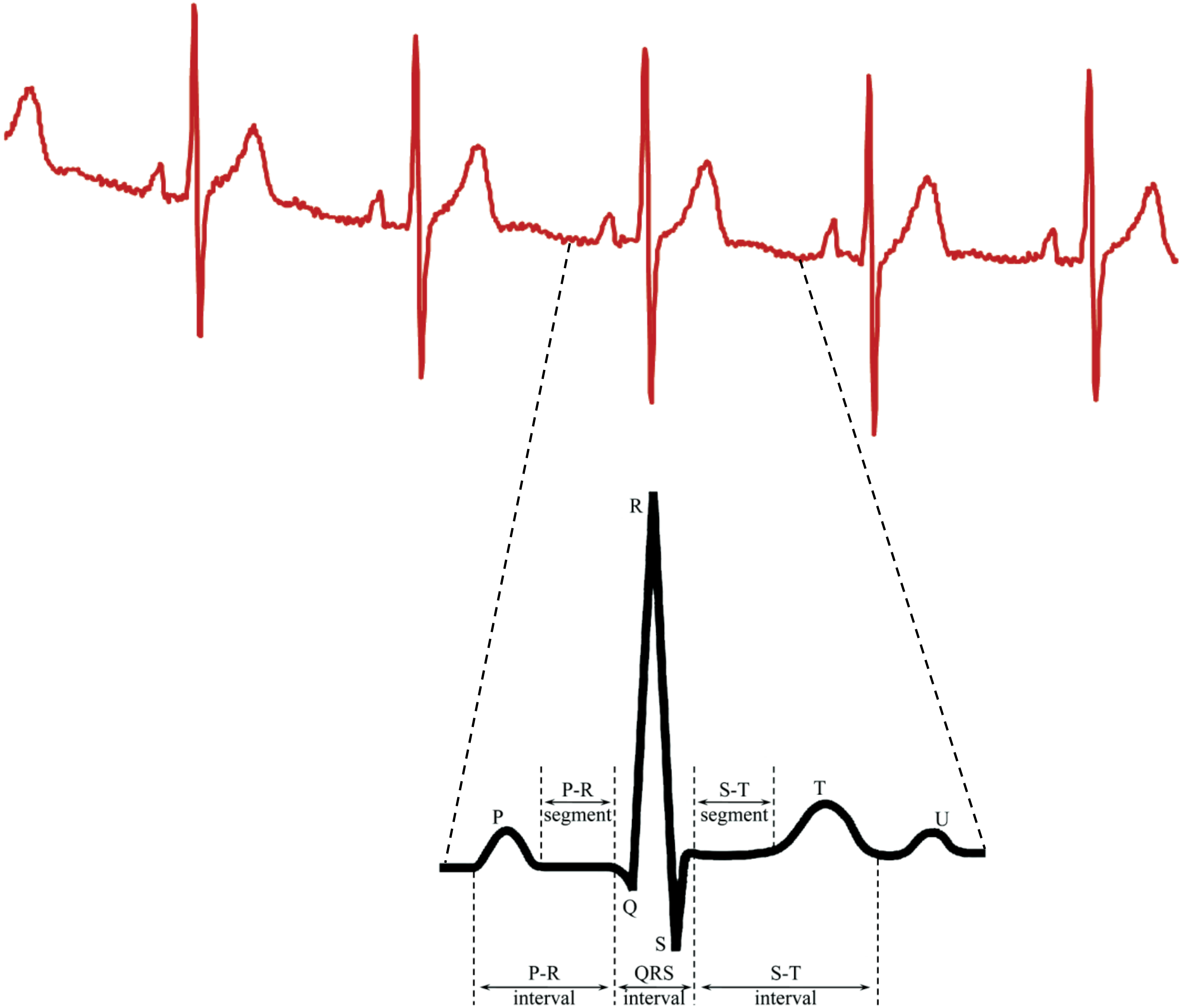
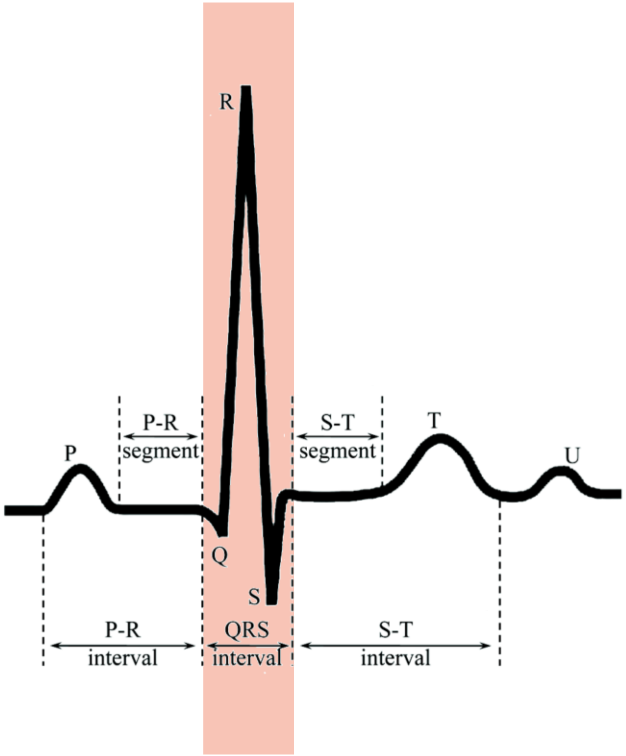
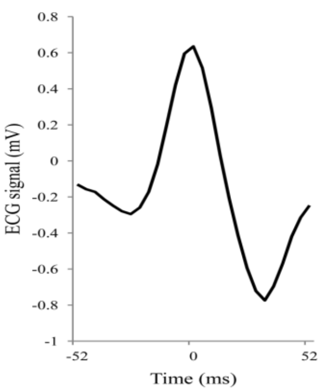
ECG signal


R
P-R
segment
S-T
segment
T
P
U
Q
S
P-R
interval
QRS
interval
S-T
interval

QRS complex (QRS interval)

- Crucial part of an ECG signal
  - Rhythm origin detection
  - Conduction abnormalities
  - Accessory pathways detection (Wolf-Parkinson-White syndrome)
  - Anti-arrhythmic medication evaluation
- Typical duration: ~100 ms
- Huge amounts of data
- Compression/reconstruction artifacts may lead to incorrect diagnosis and treatment
- *Our goal:*
  Compression and sufficient quality





Compression of ECG signals

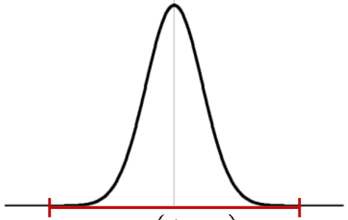
- Approaches
  - Feature extraction: duration, shape, amplitude, etc.
    [Sörnmo et al. 1981]
  - Expansion into continuous Hermite functions
    [Lo Conte et al. 1994], [Laguna et al. 1996], [Lagerholm et al. 2000]

- ***Our approach:*** expand sampled ECG signals into discrete Hermite functions
  - Obtained by properly sampling (continuous) Hermite functions

Hermite functions

- Hermite polynomials

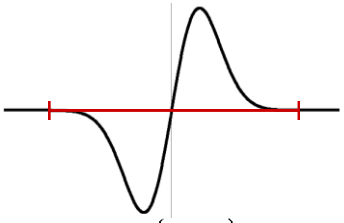
$$H_\ell(t) = 2tH_{\ell-1}(t) - 2(\ell-1)H_{\ell-2}(t)$$
$$H_0(t) = 1, \quad H_{-1}(t) = 0$$



$\varphi_0(t,\sigma)$

- Hermite functions

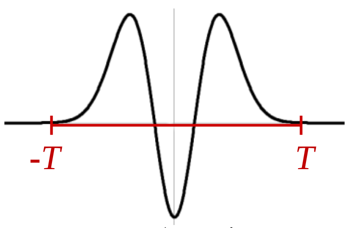
$$\varphi_\ell(t,\sigma) = \frac{e^{-t^2/2\sigma^2}}{\sqrt{\sigma 2^\ell \ell! \sqrt{\pi}}} H_\ell(t/\sigma)$$



$\varphi_1(t,\sigma)$

- Orthogonality

$$\begin{aligned} \langle \varphi_\ell, \varphi_m \rangle &= \int_{\mathbb{R}} \varphi_\ell(t,\sigma)\varphi_m(t,\sigma)dt \\ &\approx \int_{-T}^{T} \varphi_\ell(t,\sigma)\varphi_m(t,\sigma)dt = \delta_{\ell-m} \end{aligned}$$



$\varphi_2(t,\sigma)$

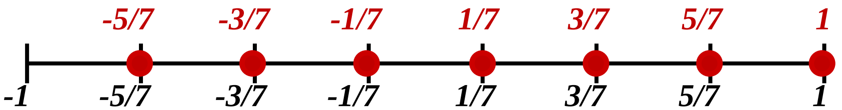
QRS compression with Hermite functions

- Motivated by visual similarity with QRS complexes
- Compression algorithm
  1. Fix σ to "stretch" or "contract" all $\varphi_\ell(t,\sigma)$
  2. Compute projection coefficients
  $$c_\ell = \int_{\mathbb{R}} s(t)\varphi_\ell(t,\sigma)dt \approx \int_{-T}^{T} s(t)\varphi_\ell(t,\sigma)dt$$
  3. Select M coefficients that achieve the tolerable error:
  $$\left|\frac{\sum_\ell c_\ell - \sum_{m=0}^{M-1} c_{\ell_m}}{\sum_\ell c_\ell}\right| \leq \epsilon$$
  4. Reconstruct the approximation:
  $$s(t) \approx \hat{s}(t) = \sum_{m=0}^{M-1} c_{\ell_m}\varphi_{\ell_m}(t,\sigma)$$

Discrete implementation

- Use numerical quadrature: $c_\ell \approx \sum_{k=-K}^{K} s(\tau_k)\varphi_\ell(\tau_k,\sigma)(t_k - t_{k-1})$
- Previously reported implementations:

$$\tau_k = t_k = \frac{Tk}{K}$$



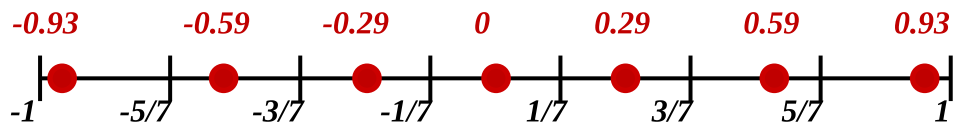
- Expansion and approximation:

$$\begin{pmatrix} c_0 & \dots & c_{2K} \end{pmatrix} = \begin{pmatrix} s(\tau_{-K}) & \dots & s(\tau_K) \end{pmatrix} \cdot \mathrm{Diag} \cdot \left[H_\ell(\frac{Tk}{K}) \right]_{0 \le k,\ell \le 2K}$$

$$\begin{pmatrix} \hat{s}(\tau_{-K}) & \dots & \hat{s}(\tau_K) \end{pmatrix} = \begin{pmatrix} \hat{c}_0 & \dots & \hat{c}_{2K} \end{pmatrix} \cdot \left[H_\ell(\frac{Tk}{K}) \right]_{0 \le k,\ell \le 2K}^{-1}$$

- Disadvantages:
  - Discretized in this manner, Hermite functions are not orthogonal
  - No efficient implementation of expansion and reconstruction

Proposed algorithm



- Sample non-equidistantly:

$$\varphi_{2K+1}(\tau_k, 1) = 0$$

$$c_\ell \approx \sum_{k=-K}^{K} s(\tau_k \lambda)\varphi_\ell(\tau_k, 1)T/K$$

- Expansion and approximation:

$$\begin{pmatrix} c_0 & \dots & c_{2K} \end{pmatrix} = \begin{pmatrix} s(\tau_{-K}) & \dots & s(\tau_K) \end{pmatrix} \cdot \mathrm{Diag} \cdot \Big[H_\ell(\tau_k \lambda) \Big]_{0 \le k, \ell \le 2K}$$

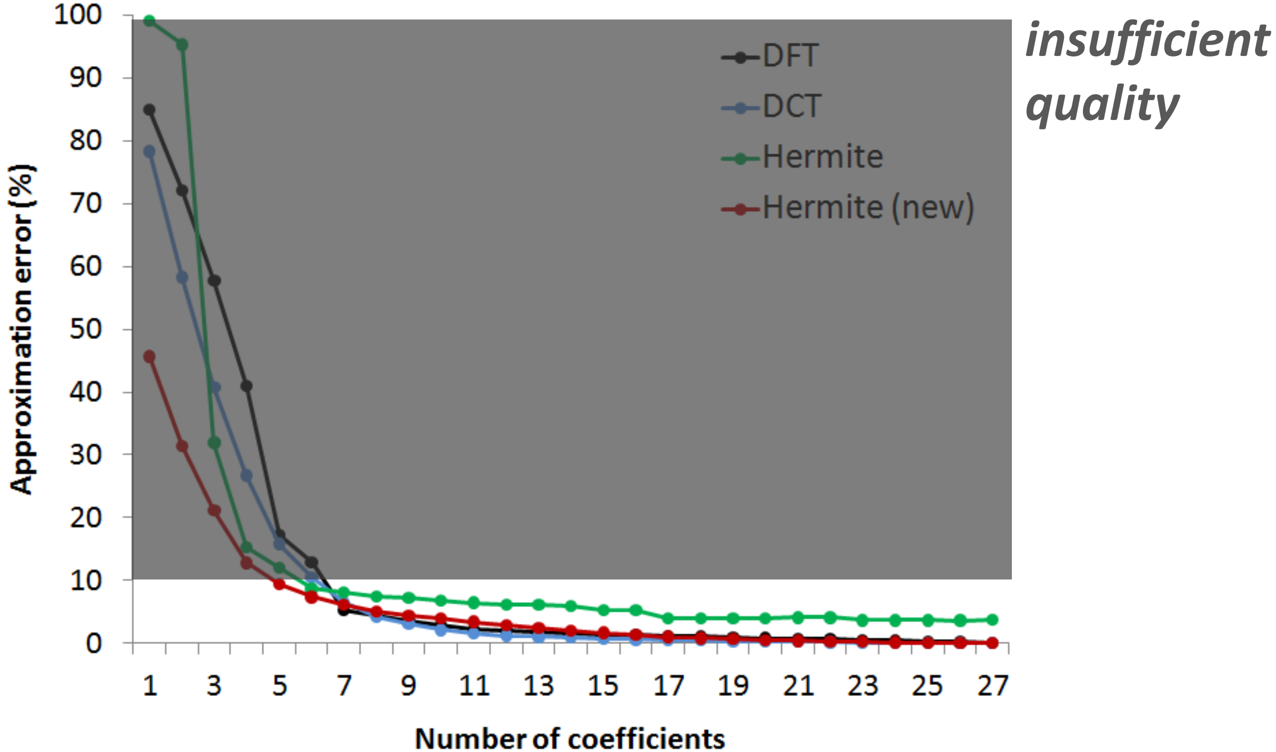
$$\begin{pmatrix} \hat{s}(\tau_{-K}) & \dots & \hat{s}(\tau_K) \end{pmatrix} = \begin{pmatrix} \hat{c}_0 & \dots & \hat{c}_{2K} \end{pmatrix} \cdot \Big[H_\ell(\tau_k \lambda) \Big]_{0 \le k, \ell \le 2K}^{T}$$

- Advantages:
  - Discretized Hermite functions are orthogonal
  - Structure: expansion and reconstruction are transposes
  - Potential efficient implementations

Experiments

- MIT-BIH ECG Compression Test Database
- ECG signals sampled at 250 Hz
- 29 QRS complexes from 3 leads
- Compare to
  - Previous algorithm (Hermite, equidistant)
  - DFT
  - DCT

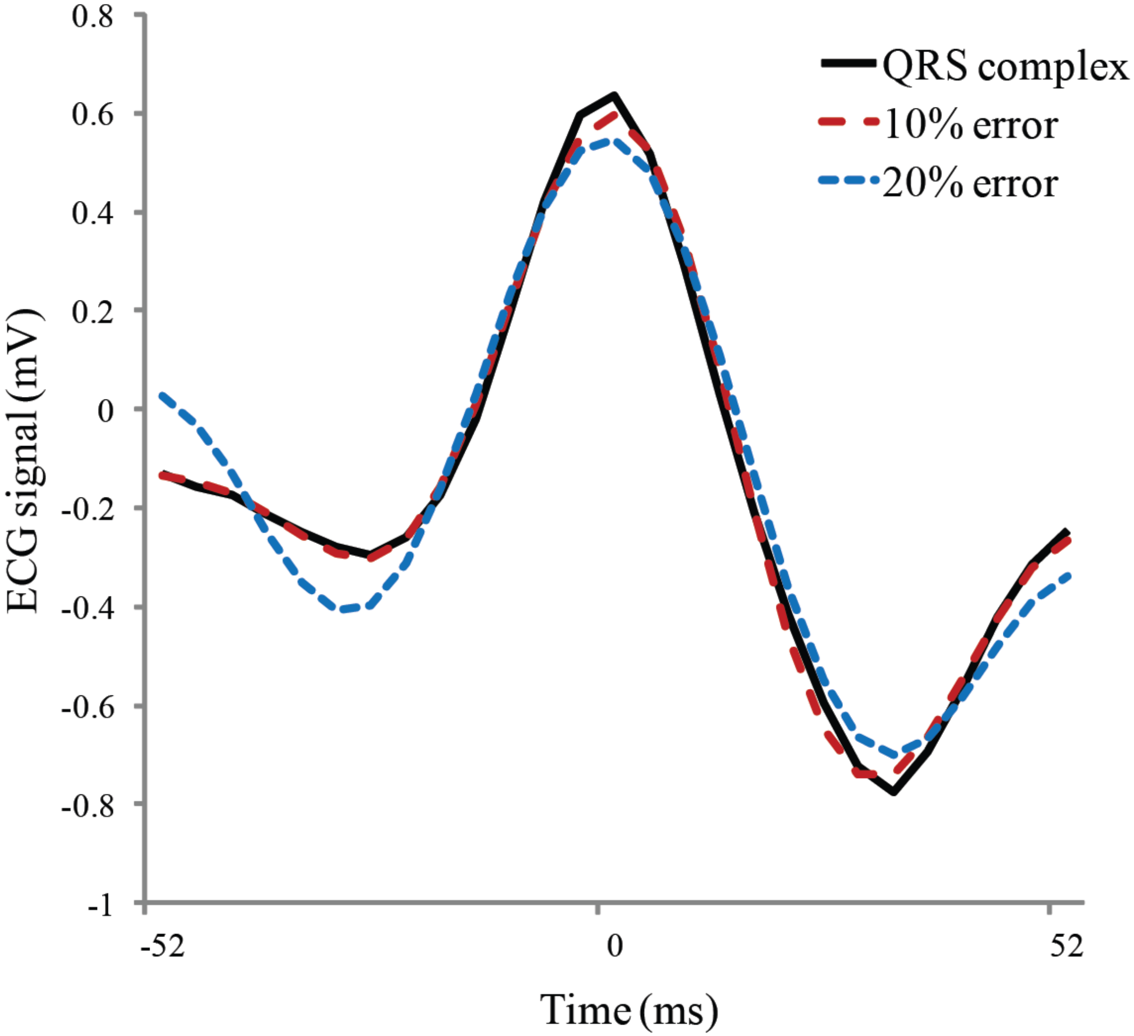
Results



Gain: about 15-20%

Compression ratio: about 5

Approximation



More recent results

- Experiment setup
  - 1486 QRS complexes from 168 leads
  - σ and λ tuned for each lead
  - Compare to previously proposed implementation, DFT, DCT, and DWT
- Results:

Error	New algorithm	Previous algorithm	DFT	DCT	DWT
10%	**5.3** (5.8)	**3.5** (9.0)	**3.7** (8.3)	**4.3** (7.3)	**3.3** (9.4)
15%	**7.0** (4.4)	**4.3** (7.2)	**4.2** (7.4)	**5.1** (6.1)	**4.2** (7.5)
20%	**9.2** (3.4)	**5.0** (6.2)	**4.6** (6.7)	**5.8** (5.3)	**4.8** (6.5)
25%	**10.4** (2.9)	**5.8** (5.4)	**5.1** (6.1)	**6.6** (4.7)	**5.5** (5.6)

Compression ratio (# of coefficients)

Conclusions

- Compression of QRS complexes
  - Using discretized Hermite functions
  - Non equidistant sampling to get orthogonality
  - Improved compression: 20–40%
- Future work
  - Adjust stretch factor for each ECG lead
  - Construct an efficient implementation
  - Extend the approach to other segments of ECG signals