P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler

Chao Lin\(^1\), Georg Kail\(^2\), Jean-Yves Tourneret\(^1\), Corinne Mailhes\(^1\), Franz Hlawatsch\(^2\)

\(^1\)University of Toulouse - IRIT/ENSEEIHT/TéSA, Toulouse, FRANCE
\(^2\)Vienna University of Technology - Institute of Telecommunications, Vienna, AUSTRIA

IEEE ICASSP’11, Prague, Czech Republic
Outline

1. **Context**
   Electrocardiogram (ECG)
   ECG delineation

2. **Problem formulation**

3. **Bayesian model**
   Likelihood function
   Prior distribution

4. **Block Gibbs sampler**

5. **Results**
   Typical examples
   Evaluation on QT database

6. **Conclusion and future works**

7. **Appendix**
Outline

1 Context
Electrocardiogram (ECG)
ECG delineation

2 Problem formulation

3 Bayesian model
Likelihood function
Prior distribution

4 Block Gibbs sampler

5 Results
Typical examples
Evaluation on QT database

6 Conclusion and future works

7 Appendix
Electrocardiogram (ECG)

- A recording of the electrical activity of the heart over time
- 3 distinct waves are produced during cardiac cycle
  - P wave caused by atrial depolarization
  - QRS complex caused by ventricular depolarization
  - T wave results from ventricular repolarization and relax
- Wave shapes and interval durations indicate clinically useful information
ECG delineation

- Delineation: determination of peaks and boundaries of the waves
ECG delineation

- Delineation: determination of peaks and boundaries of the waves
- P and T wave delineation—an challenging problem
Context

ECG delineation

- Delineation: determination of peaks and boundaries of the waves
- P and T wave delineation—a challenging problem
- Existing methods
  - **Filtering**: adaptive filtering, nested median filtering, low pass differentiation (LPD)
  - **Basis expansions**: Fourier transform, discrete cosine transform, wavelet transform (WT)
  - **Classification and pattern recognition**: fuzzy theory, hidden Markov models, artificial neural networks
  - **Bayesian inference**: off line and sequential approaches (Kalman filter)
Problem formulation

Outline

1. Context
   Electrocardiogram (ECG)
   ECG delineation

2. Problem formulation

3. Bayesian model
   Likelihood function
   Prior distribution

4. Block Gibbs sampler

5. Results
   Typical examples
   Evaluation on QT database

6. Conclusion and future works

7. Appendix
Problem formulation

Signal model for the non-QRS intervals

- \( N_{T,1} \) and \( N_{P,1} \) for the first non-QRS interval
- \( N_{T,D} \) and \( N_{P,D} \) for the last non-QRS interval

(a)

<table>
<thead>
<tr>
<th>QRS1</th>
<th>I_{T,1}</th>
<th>I_{P,1}</th>
<th>QRS2</th>
<th>\ldots</th>
</tr>
</thead>
</table>

(b)

\( I_1 \) and \( I_D \) for the first and last non-QRS intervals

(c)

\( \ldots \) for the remaining non-QRS intervals

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Problem formulation

Signal model for the non-QRS intervals

(a) QRS

(b) T, P

(c) a_{T,i}, a_{P,j}, a_{T,m}, a_{P,n}

b_{T,i} = 1, b_{P,j} = 1, b_{T,m} = 1, b_{P,n} = 1
Problem formulation

Signal model for the non-QRS intervals

non-QRS signal components within a $D$-beat window

$$x_k = \sum_{l=-L/2}^{L/2} h_{T,l} u_{T,k-l} + \sum_{l=-L/2}^{L/2} h_{P,l} u_{P,k-l} + c_k + w_k, \ k \in J^*, \ (1)$$

- $u_{T,k} = b_{T,k} a_{T,k}$: unknown “impulse” sequence indicating T wave locations and amplitudes,
- $u_{P,k} = b_{P,k} a_{P,k}$: unknown “impulse” sequence indicating P wave locations and amplitudes,
- $h_T = (h_{T,-L/2} \cdots h_{T,L/2})^T$: unknown T waveform,
- $h_P = (h_{P,-L/2} \cdots h_{P,L/2})^T$: unknown P waveform,
- $c_k$: baseline sequence, $w_k$: white Gaussian noise
Problem formulation

Signal model for the non-QRS intervals

- Representation of the P and T waveforms by a Hermite basis expansion
  \[ h_T = H \alpha_T , \quad h_P = H \alpha_P , \]
  - \( H \) is a \((L + 1) \times G\) matrix whose columns are the first \( G \) Hermite functions with \( G \leq (L + 1) \)
  - \( \alpha_T \) and \( \alpha_P \) are unknown coefficient vectors of length \( G \)

- Modeling of the local baseline within the \( n \)-th non-QRS interval by a 4th-degree polynomial
  \[ c_n = M_n \gamma_n , \]
  - \( M_n \) is the known \( N_n \times 5 \) Vandermonde matrix
  - \( \gamma_n = (\gamma_{n,1} \cdots \gamma_{n,5})^T \) is the unknown coefficient vector
Signal model for the non-QRS intervals

Vector representation of the non-QRS signal in (1)

\[ x = F_T B_T a_T + F_P B_P a_P + M \gamma + w, \quad (2) \]

- \( b_T, b_P, a_T, \) and \( a_P \) denote the \( M \times 1 \) vectors corresponding to \( b_{T,k}, b_{P,k}, a_{T,k}, \) and \( a_{P,k} \), respectively.
- \( B_T \triangleq \text{diag}(b_T) \) and \( B_P \triangleq \text{diag}(b_P) \),
- \( F_T \) and \( F_P \) are the \( K \times M \) Toeplitz matrices with first row \((h_1^T \alpha_T \ 0_{M-1}^T)\) and \((h_1^T \alpha_P \ 0_{M-1}^T)\), respectively.
- \( M \), and \( \gamma \) are obtained by concatenating the \( M_n \) and \( \gamma_n \), for \( n = 1, \ldots, D \).
Outline

1 Context
   Electrocardiogram (ECG)
   ECG delineation

2 Problem formulation

3 Bayesian model
   Likelihood function
   Prior distribution

4 Block Gibbs sampler

5 Results
   Typical examples
   Evaluation on QT database

6 Conclusion and future works

7 Appendix
Bayesian model

**Model parameters**

Bayesian estimation relies on the posterior distribution

\[ p(\theta|x) \propto p(x|\theta)p(\theta) \]

\[ \theta = \left( \theta_T^T \theta_P^T \theta_{cw}^T \right)^T \]

are the unknown parameters resulting from (2)

- \( \theta_T \triangleq (b_T^T a_T^T \alpha_T^T)^T \) and \( \theta_P \triangleq (b_P^T a_P^T \alpha_P^T)^T \) are T and P wave related parameter vectors,
- \( \theta_{cw} \triangleq (\gamma^T \sigma_w^2)^T \) are baseline and noise parameters.

**Likelihood function**

\[
p(x|\theta) \propto \frac{1}{\sigma_K} \exp\left(-\frac{1}{2\sigma_w^2} \|x - F_T B_T a_T - F_P B_P a_P - M\gamma\|^2\right),
\]

where \( \| \cdot \| \) is the \( \ell_2 \) norm, i.e., \( \|x\|^2 = x^T x \).
Bayesian model

**Location prior**

**T wave indicator prior: block constraint**

\[
p(b_{J_{T,n}}) = \begin{cases} 
  p_0 & \text{if } \|b_{J_{T,n}}\| = 0 \\
  p_1 & \text{if } \|b_{J_{T,n}}\| = 1 \\
  0 & \text{otherwise,}
\end{cases}
\]

Assuming independence between consecutive non-QRS intervals, the prior of \(b_T\) is given by

\[
p(b_T) = D \prod_{n=1}^{D} p(b_{J_{T,n}}).
\]
Bayesian model

### Amplitude and waveform priors

#### T wave amplitude prior

\[ p(a_{T,k} | b_{T,k} = 1) = \mathcal{N}(a_{T,k}; 0, \sigma_a^2) \]

- \( a_{T,k} \) are only defined at time instants \( k \) where \( b_{T,k} = 1 \),
- \( u_{T,k} = b_{T,k} a_{T,k} \) is a Bernoulli-Gaussian sequence with block constraints.

#### T waveform coefficients prior

\[ p(\alpha_T) = \mathcal{N}(\alpha_T; 0, \sigma_\alpha^2 I_{L+1}) \]

The priors of the P wave parameters \( b_P, a_P \) and \( \alpha_P \) are defined in a fully analogous way!

---

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Bayesian model

Baseline and noise variance priors

Baseline coefficient prior

\[ p(\gamma) = \mathcal{N}(\gamma; 0, \sigma_{\gamma}^2 I_{5D}) \]

Noise variance prior

\[ \sigma_{w}^2 = IG(\sigma_w^2; \xi, \eta) \]

Conjugate priors for simplicity

Posterior distribution

\[ p(\theta|x) \propto p(x|\theta)p(\theta) = p(x|\theta)p(\theta_T)p(\theta_P)p(\theta_{cw}) \]

Complex distribution
Outline

1 Context
   Electrocardiogram (ECG)
   ECG delineation

2 Problem formulation

3 Bayesian model
   Likelihood function
   Prior distribution

4 Block Gibbs sampler

5 Results
   Typical examples
   Evaluation on QT database

6 Conclusion and future works

7 Appendix
- In a $D$-beat processing window, for each non-QRS interval:
  - Sample the $T$ indicator block $b_{JT,n}$
  - For the $k$ where $b_{T,k} = 1$, sample the $T$ amplitudes $a_{T,k}$
  - Sample the $P$ indicator block $b_{JP,n}$
  - For the $k$ where $b_{P,k} = 1$, sample the $P$ amplitudes $a_{P,k}$

- Sample $P$ and $T$ waveform coefficients $\alpha_T$ and $\alpha_P$
- Sample baseline coefficients $\gamma$
- Sample noise variance $\sigma_w^2$
Results

Outline

1 Context
   Electrocardiogram (ECG)
   ECG delineation

2 Problem formulation

3 Bayesian model
   Likelihood function
   Prior distribution

4 Block Gibbs sampler

5 Results
   Typical examples
   Evaluation on QT database

6 Conclusion and future works

7 Appendix
Results

Simulation parameters

- Processing window length: $D = 10$,
- For each estimation, the 40 first iterations are disregarded (burn-in period) and 60 iterations are used to compute the estimates.
- Real ECG datasets from the QT database.
- Computation time: 8 seconds to run 100 iterations on a 10-beat ECG block (Matlab implementation).

Results

Typical examples

ECG signal: dataset sele0136

Posterior distributions of the P and T−wave indicator locations

Samples

P−waveform estimation (normalized)

Samples

T−waveform estimation (normalized)
Results

Typical examples

ECG signal: dataset sele0136

Posterior distributions of the P and T−wave indicator locations

T−waveform estimation (normalized)
P−waveform estimation (normalized)
Results

Typical examples

ECG signal: dataset sele0136

Posterior distributions of the P and T–wave indicator locations

T–waveform estimation (normalized)

P–waveform estimation (normalized)
Results

Typical examples

(a) ECG signal: dataset sele0136

(b) Posterior distributions of the P and T−wave indicator locations

(c) T−waveform estimation (normalized)

(d) P−waveform estimation (normalized)

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Results

Typical examples

ECG signal: dataset sele0136

Posterior distributions of the P and T-wave indicator locations

T-waveform estimation (normalized)

P-waveform estimation (normalized)
Results

Typical examples

(a)

(b)
Results

Typical examples

(a) time (s)

(b) time (s)
Results

Typical examples

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Results

Typical examples

(a) ECG signal, estimated P and T waves, estimated baseline

(b) Time (s)
Results

Premature ventricular contraction ECG

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Results

Premature ventricular contraction ECG

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Biphasic T wave ECG

ECG signal: dataset sele0607

Posterior distributions of the P and T−wave indicator locations

T−waveform estimation (normalized)

P−waveform estimation (normalized)
Results

Biphasic T wave ECG

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Absence of P wave

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Results

Absence of P wave

![ECG signal with estimated P and T-waves and baseline](image)

- ECG signal
- estimated P and T-waves
- estimated baseline

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
### Evaluation on QTDB

Table: Delineation and detection performance for QTDB

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Proposed alg.</th>
<th>LPD</th>
<th>WT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_P$: Se (%)</td>
<td>99.60</td>
<td>97.70</td>
<td>98.87</td>
</tr>
<tr>
<td>Onset-P: $\mu \pm \sigma$ (ms)</td>
<td>1.7 ±10.8</td>
<td>14.0 ±13.3</td>
<td>2.0 ±14.8</td>
</tr>
<tr>
<td>Peak-P: $\mu \pm \sigma$ (ms)</td>
<td>2.7 ±8.1</td>
<td>4.8 ±10.6</td>
<td>3.6 ±13.2</td>
</tr>
<tr>
<td>End-P: $\mu \pm \sigma$ (ms)</td>
<td>2.5 ±11.2</td>
<td>−0.1 ±12.3</td>
<td>1.9 ±12.8</td>
</tr>
<tr>
<td>$b_T$: Se (%)</td>
<td>100</td>
<td>97.74</td>
<td>99.77</td>
</tr>
<tr>
<td>Onset-T: $\mu \pm \sigma$ (ms)</td>
<td>5.7 ±16.5</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Peak-T: $\mu \pm \sigma$ (ms)</td>
<td>0.7 ±9.6</td>
<td>−7.2 ±14.3</td>
<td>0.2 ±13.9</td>
</tr>
<tr>
<td>End-T: $\mu \pm \sigma$ (ms)</td>
<td>2.7 ±13.5</td>
<td>13.5 ±27.0</td>
<td>−1.6 ±18.1</td>
</tr>
</tbody>
</table>
Conclusion and future works

Outline

1 Context
   Electrocardiogram (ECG)
   ECG delineation

2 Problem formulation

3 Bayesian model
   Likelihood function
   Prior distribution

4 Block Gibbs sampler

5 Results
   Typical examples
   Evaluation on QT database

6 Conclusion and future works

7 Appendix
Conclusion and future works

Conclusion

- A Bayesian model for the non-QRS intervals of ECG signals,
- A block Gibbs sampler for joint delineation and waveform estimation of P and T waves,
- Evaluation on the QTDB is promising.

Prospects

- Exploitation of the amplitude estimation, ex., TWA detection,
- Exploitation of the waveform estimation, ex., arrhythmia detection,
- Beat-to-beat / sequential delineation.
Thank you for your attention!

Matlab demo available at
http://www.enseeiht.fr/~lin
Appendix

Outline

1. Context
   Electrocardiogram (ECG)
   ECG delineation

2. Problem formulation

3. Bayesian model
   Likelihood function
   Prior distribution

4. Block Gibbs sampler

5. Results
   Typical examples
   Evaluation on QT database

6. Conclusion and future works

7. Appendix

P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler
Appendix

Time-shift and scale ambiguities

Issue: No unique solution for a convolution model

- Scale ambiguity: \( h \star u = (ah) \star (u/a), \quad \forall a \neq 0, \)
- Time-shift ambiguity: \( h \star u = (d_{\tau} \star h) \star (d_{-\tau} \star u), \quad \forall \tau \in \mathbb{Z}. \)

Solution: Hybrid Gibbs sampling

- Metropolis-Hastings within Gibbs after sampling waveform coefficients,
- Deterministic shifts after sampling waveform coefficients:
  - Time-shifts to have \( h'_0 = \max |h|, \)
  - Scale-shifts to have \( h'_0 = 1, \)

Appendix

References

Similar applications on physiological signal processing:

- **on OCT signals:** G. Kail et al., A blind Monte Carlo detection-estimation method for optical coherence tomography, *ICASSP*, 2009

Other P and T wave delineation methods:

- **KF:** O. Sayadi et al., A model-based Bayesian framework for ECG beat segmentation, *J. Physiol. Meas.*, 2009