DOA estimation with a vector-sensor in the presence of noise

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May 27, 2011

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Motivation

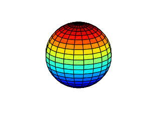
- Conventional methods for DOA estimation involve multiple microphones and entail spacing requirements.
- Compact superdirective devices have been utilized for DOA estimation.
- Vector sensors are inherently superdirective devices.
- We propose an improved method to estimate DOA using a single vector-sensor.

What is a vector-sensor?

Conventional microphone:

Channels: 1

Directivity: monopole (*typically*)

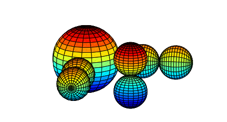


Schematic:

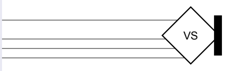
Vector-sensor:

Channels: 4

Directivity: monopole ($\times 1$)
dipole ($\times 3$)



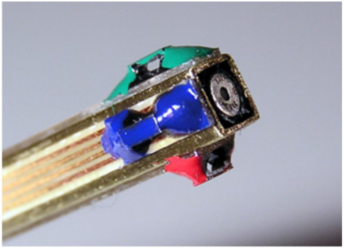
Schematic:



Construction of vector-sensor

Dipole elements may be obtained from:

1. Particle velocity sensors
2. Differential-microphone arrays

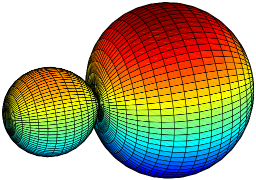


Notation for measurements

The measurements of the vector-sensor are denoted:

$$\mathbf{y}[n] = \begin{bmatrix} p[n] \\ v_x[n] \\ v_y[n] \\ v_z[n] \end{bmatrix} = \begin{bmatrix} p[n] \\ \mathbf{v}[n] \end{bmatrix}$$

A linear combination of the sensor signals produces a limaçon.

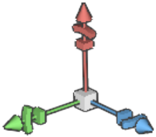


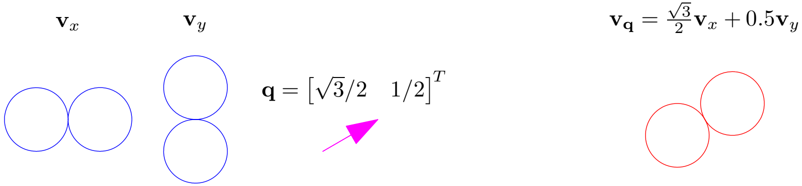
Scenario details

- The source is located in the far-field.
- The signal produced is denoted $s[n]$.
- The DOA is described by a unit vector $\mathbf{u}$.
- Noise components are assumed to be uncorrelated which applies to sensor-noise and isotropic fields.

Steering

- The orientation of the dipole sensors are fixed.
- A linear combination of these sensors allows for steering in any direction $\mathbf{q}$.
- $v_{\mathbf{q}}[n] = \mathbf{q}^T \mathbf{v}[n]$





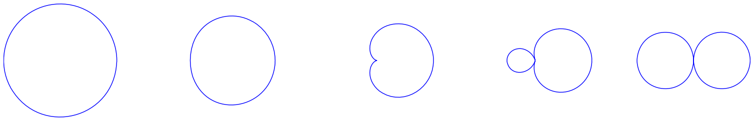
Beam shaping

- A weighted combination of monopole and steered dipole produces a limaçon pattern.
- The combination

$$y_{\mathbf{q}}[n] = \alpha p[n] + (1 - \alpha) v_{\mathbf{q}}[n] \,.$$

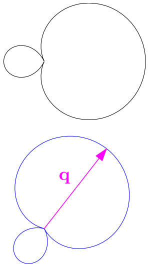
ensures unity boresight response.

$\alpha = \ldots$ 1 0.75 0.5 0.25 0



Steered response power (SRP)

1. We choose a beampattern paramater α.
2. We steer the beam in the direction $\mathbf{q}$.
3. We measure the response power over N samples.



The SRP is defined as the average power:

$$\mathrm{SRP}(\alpha, \mathbf{q}) = \frac{1}{N} \sum_{n=0}^{N-1} \Big(\alpha p[n] + (1-\alpha) \mathbf{q}^T \mathbf{v}[n] \Big)^2 .$$

DOA estimation

For a given α we search for the direction $\mathbf{q}$ corresponding to maximum SRP.

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\text{argmax}} \{\mathrm{SRP}(\alpha, \mathbf{q})\}$$
$$\text{subject to} \quad \mathbf{q}^T \mathbf{q} = 1\,.$$

Simplified form

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\operatorname{argmax}} \left\{ \alpha\, \mathbf{q}^T \hat{\mathbf{r}}_{pv} + \frac{1-\alpha}{2} \mathbf{q}^T \hat{\mathbf{R}}_{vv} \mathbf{q} \right\}$$
$$\text{subject to} \quad \mathbf{q}^T \mathbf{q} = 1\,,$$

where:

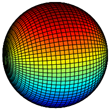
$$\hat{\mathbf{R}}_{vv} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{v}[n] \mathbf{v}^T[n]$$
$$\hat{\mathbf{r}}_{pv} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbf{v}[n] p[n]\,.$$

Let's inspect two extreme cases...

Case I: Near-monopole

- For a near-monopole beampattern ($\alpha \to 1^-$), the problem becomes:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\operatorname{argmax}}\ \mathbf{q}^T \hat{\mathbf{r}}_{pv}$$
$$\text{s.t.} \quad \mathbf{q}^T \mathbf{q} = 1 \,.$$



- The solution is:

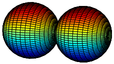
$$\hat{\mathbf{u}} = \frac{\hat{\mathbf{r}}_{pv}}{\|\hat{\mathbf{r}}_{pv}\|} \,.$$

Note: This estimator was proposed by Davies ('87) and later by Nehorai and Paldi ('94) under the name "*Intensity-Based Algorithm*".

Case II: Dipole

- For a dipole beampattern ($\alpha = 0$), the problem becomes:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\text{argmax}}\ \mathbf{q}^T \hat{\mathbf{R}}_{vv} \mathbf{q}$$
$$\text{s.t.} \quad \mathbf{q}^T \mathbf{q} = 1\,.$$



- The solution is:

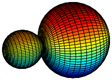
$$\hat{\mathbf{u}} = \text{eigenvector corresponding to largest eigenvaue of } \hat{\mathbf{R}}_{vv}\,.$$

Note: This estimator was proposed by Nehorai and Paldi ('94).

General case: Limaçon beampattern

- The general case ($0 \leq \alpha < 1$), the problem involves both linear and quadratic terms:

$$\hat{\mathbf{u}} = \underset{\mathbf{q}}{\text{argmax}} \left\{ \alpha\, \mathbf{q}^T \hat{\mathbf{r}}_{pv} + \frac{1-\alpha}{2} \mathbf{q}^T \hat{\mathbf{R}}_{vv} \mathbf{q} \right\}$$
$$\text{subject to} \quad \mathbf{q}^T \mathbf{q} = 1 \,,$$



- We propose an iterative gradient based solution. The gradient of the target function is:

$$\nabla_{\mathbf{q}} T(\mathbf{q}) = \alpha\, \hat{\mathbf{r}}_{pv} + (1-\alpha) \hat{\mathbf{R}}_{vv} \mathbf{q} \,,$$

Algorithm

Constrained gradient ascent algorithm for SRP maximization

Input: $\hat{\mathbf{R}}_{vv}$, $\hat{\mathbf{r}}_{pv}$, α

$\mathbf{q}_0 := \hat{\mathbf{u}} = \hat{\mathbf{r}}_{pv}/\|\hat{\mathbf{r}}_{pv}\|$

$k := 0$

$K :=$ maximum number of iterations

$\epsilon :=$ tolerance parameter

$\mu :=$ step size parameter

repeat

$\quad \mathbf{q}_{k+1} := \mathbf{q}_k + \mu\big(\alpha\,\hat{\mathbf{r}}_{pv} + (1-\alpha)\hat{\mathbf{R}}_{vv}\mathbf{q}_k\big)$

$\quad \mathbf{q}_{k+1} := \mathbf{q}_{k+1}/\|\mathbf{q}_{k+1}\|$

$\quad k := k+1$

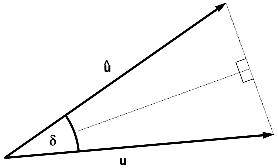
until $(k = K)$ *or alternatively* $(\|\mathbf{q}_k - \mathbf{q}_{k-1}\|^2 < \epsilon^2)$

Output: $\hat{\mathbf{u}} := \mathbf{q}_k$

Mean square angular error

- The *angular error* is the angle δ by which $\hat{\mathbf{u}}$ deviates from $\mathbf{u}$, defined formally as:

$$\mathrm{AE} \equiv 2\sin^{-1}\left(\frac{\|\hat{\mathbf{u}}-\mathbf{u}\|}{2}\right).$$



- The *mean square angular error* (MSAE) describes convergence rate of $\hat{\mathbf{u}}$ towards the true DOA:

$$\mathrm{MSAE} \equiv \lim_{N\to\infty}\left(N \cdot E\{\mathrm{AE}^2\}\right).$$

Monte Carlo simulation test

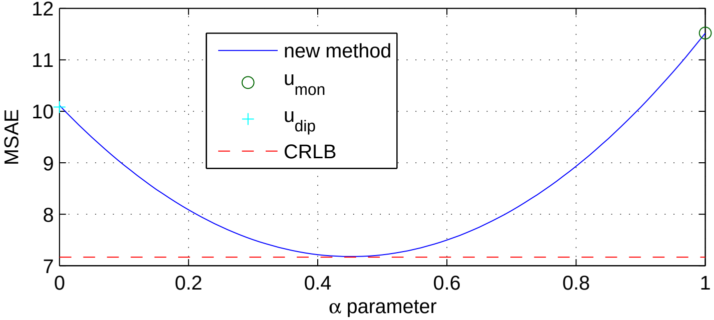
Steps for single Monte Carlo trial

1. A DOA $\mathbf{u}$ is selected randomly.
2. The signals and noise components are generated ($N = 8000$ time instants), and are used to produce sensor measurements $p[n]$, $\mathbf{v}[n]$.
3. The estimators $\hat{\mathbf{u}}$ are calculated for values of α ranging from 0 to 1; the corresponding angular errors are recorded.

A total of $MC = 100,000$ independently conducted trials creates records from which the sample MSAEs are obtained.

Monte Carlo results

Signal and noise power: $\sigma_s^2 = 0.5$, $\sigma_{e_p}^2 = 1.1$, $\sigma_{e_v}^2 = 0.9$:



Note: the estimator $\hat{\mathbf{u}}$ approaches the Cramér-Rao lower bound.

Conclusions

- A single vector-sensor provides direction-sensitive information.
- We derive a method for DOA estimation in the presence of noise.
- The method generalizes two previously suggested methods.
- The proposed method attains lower MSAE than previously suggested methods.