Design of robust steerable broadband beamformers incorporating microphone gain and phase error characteristics

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Outline

- Problem statement
- Array geometry
- Steerable broadband beamformer (SBBF) structure
- Robust design formulation
- Design example and results
- Conclusion

Problem statement

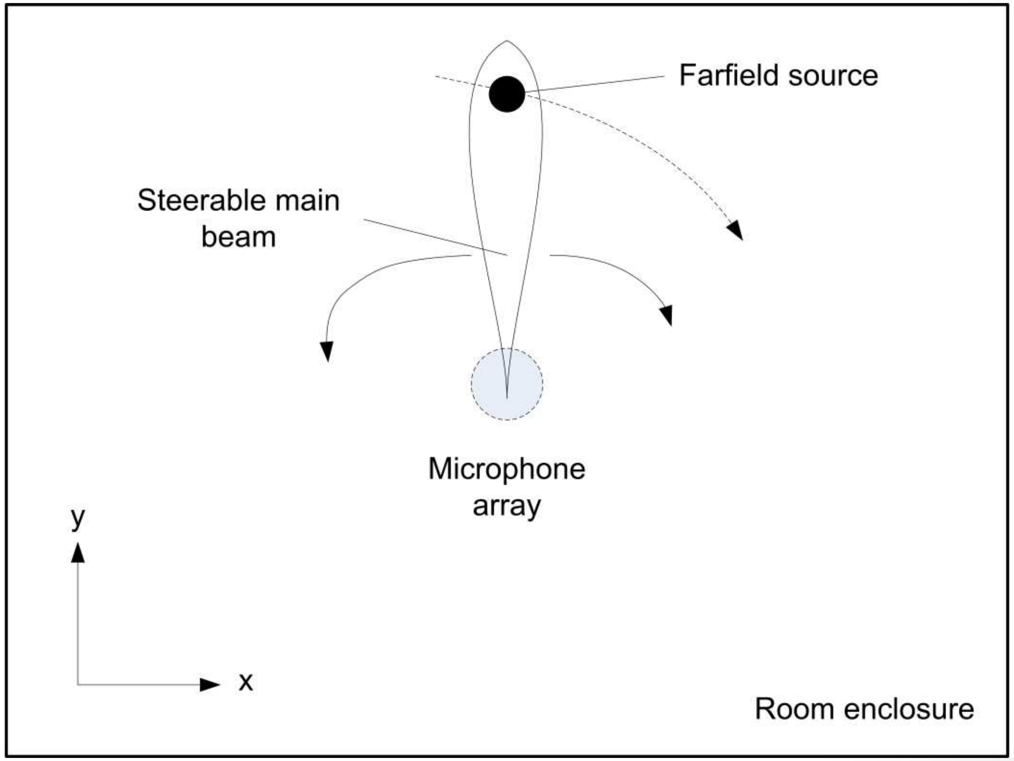
- In speech acquisition applications (e.g. tele-conferencing, audio surveillance) it is desired to have:

a) Steerable beamformer

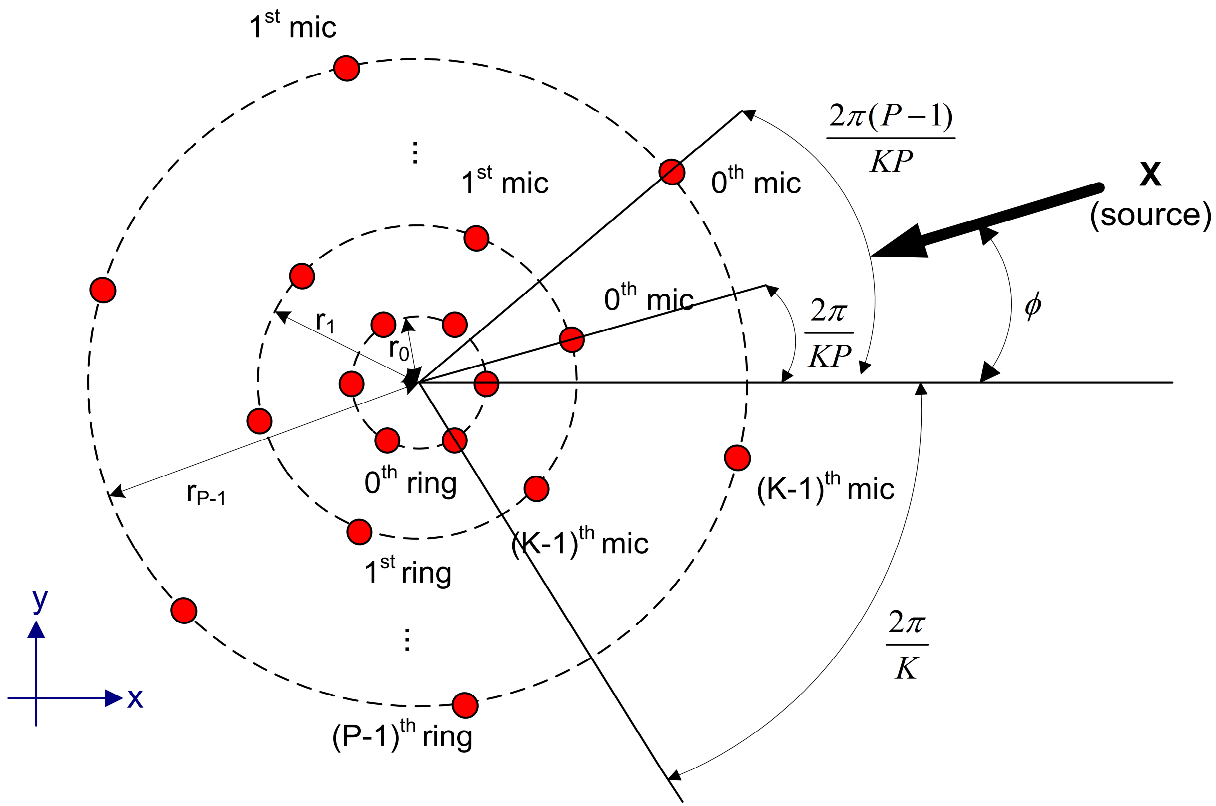
b) Frequency invariant property (broadband)

c) Robust to microphones errors and deviations

- Discussion in farfield model in azimuth plane



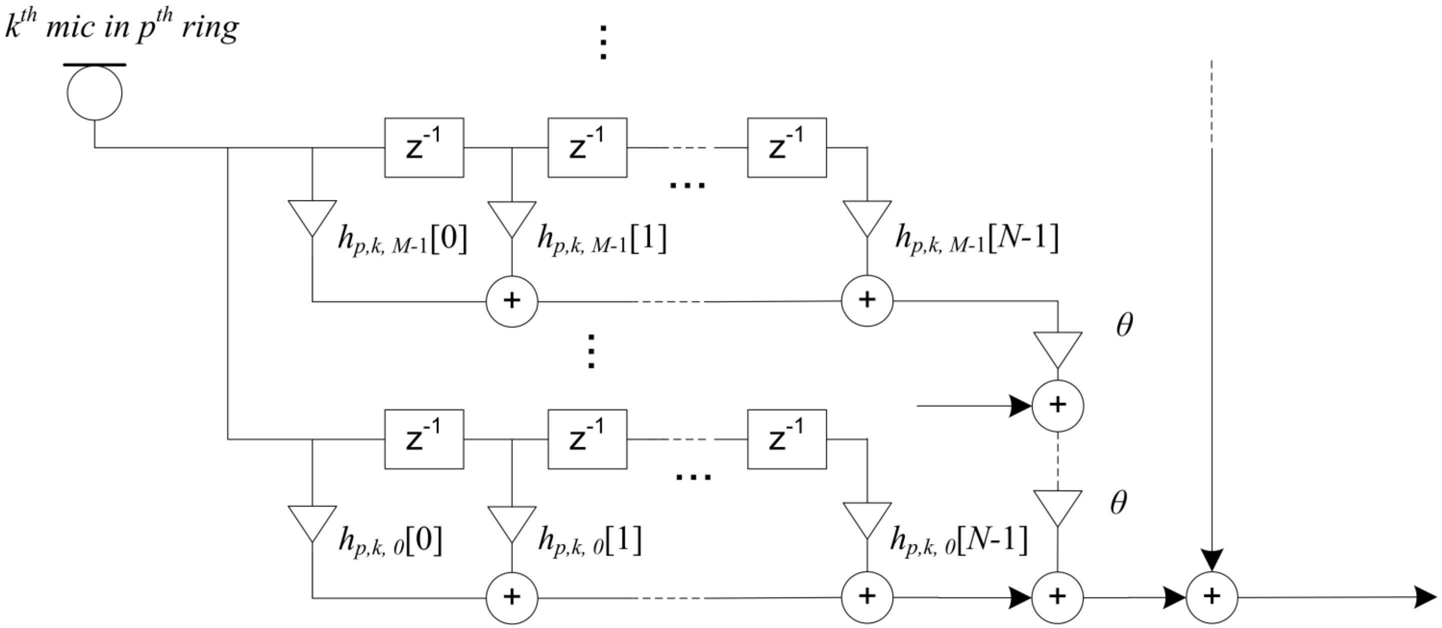
Array geometry



- Multi-ring, con-centric, circular symmetry
- Element response (farfield and free-field)

$$d_{p,k}(\omega,\phi) = \exp\left(j\frac{\omega r_p}{c}\cos(\phi - \frac{2\pi k}{K} - \frac{2\pi p}{KP}) \right)$$

SBBF structure



- Using Farrow structures at each microphone
- Steering variable: $\theta = \psi / (2\psi_{\max})$
- Nominal Beampattern:

$$G(\psi,\omega,\phi) = \sum_{p}\sum_{k}\sum_{m}\sum_{n} h_{p,k,m}[n] d_{p,k}(\omega,\phi) \left(\psi / (2\psi_{\max})\right)^{m} \exp(-j\omega n)$$

$$= \mathbf{a}^{\mathrm{H}}(\psi,\omega,\phi)\mathbf{h}$$

where $\mathbf{a}(\psi,\omega,\phi)$ and $\mathbf{h}$ are PKMNx1 long vector

Impact on model errors

- In practice deviation from ideal model in microphone elements can occur from:
  a) microphone mismatch
  b) non-ideal characteristics of microphone
  c) microphone position error
  d) local scattering effects

- Results in perturbed element response

$$\hat{d}_{p,k}(\omega,\phi) = \kappa_{p,k}(\omega,\phi)\exp\left(j\gamma_{p,k}(\omega,\phi)\right)d_{p,k}(\omega,\phi)$$

- The beampattern with perturbed element response is

$$\hat{G}(\psi,\omega,\phi) = \hat{\mathbf{a}}^{\mathrm{H}}(\psi,\omega,\phi)\mathbf{h}$$

Robust design

- Robust design based on mean of deviation, i.e.

$$E\left\{\kappa_k(\omega,\phi)\exp\left(j\gamma_k(\omega,\phi)\right)\right\}$$

- Least squares cost function

$$J = \int_{\Psi}\int_{\Omega}\int_{\Phi}\left|\hat{G}(\psi,\omega,\phi) - H_d(\psi,\omega,\phi)\right|^2 d\phi d\omega d\psi$$

$$= \mathbf{h}^{\mathrm{H}}\mathbf{Q}\mathbf{h} - 2\,\mathrm{Re}\left\{\mathbf{b}^{\mathrm{H}}\mathbf{h}\right\} + d$$

where

$$\mathbf{Q} = \int_{\Psi}\int_{\Omega}\int_{\Phi}\bar{\mathbf{Q}}_{\delta}(\psi,\omega,\phi) \otimes \mathbf{Q}_o(\psi,\omega,\phi)d\phi d\omega d\psi$$

$$\mathbf{b} = \int_{\Psi}\int_{\Omega}\int_{\Phi}\bar{b}_{\delta}(\psi,\omega,\phi)\mathbf{b}_o(\psi,\omega,\phi)d\phi d\omega d\psi$$

$$d = \int_{\Psi}\int_{\Omega}\int_{\Phi}\left|H_d(\psi,\omega,\phi)\right|^2 d\phi d\omega d\psi$$

- The mean error terms are embedded in **Q** and **b**

Design example

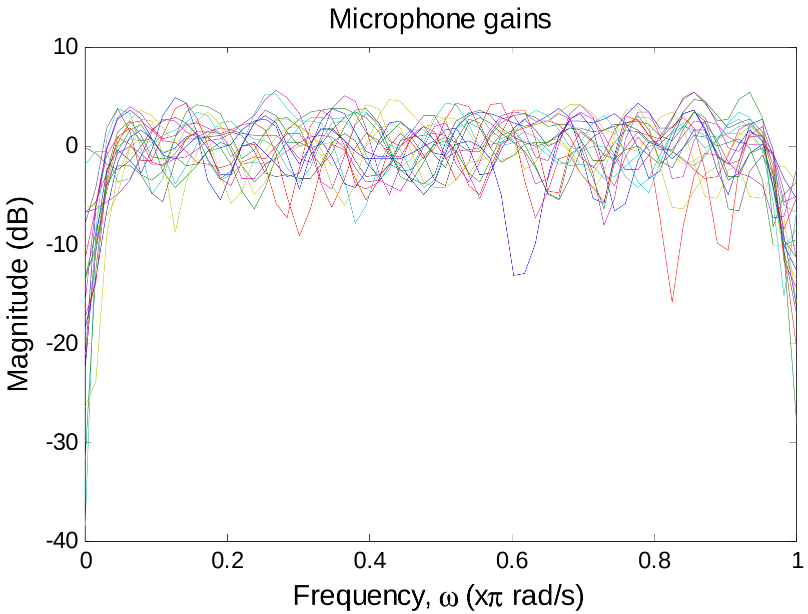
- LS formulation: min $J = \mathbf{h}^{\mathrm{H}}\mathbf{Q}\mathbf{h} - 2\,\mathrm{Re}\{\mathbf{b}^{\mathrm{H}}\mathbf{h}\} + d$
- Specification:

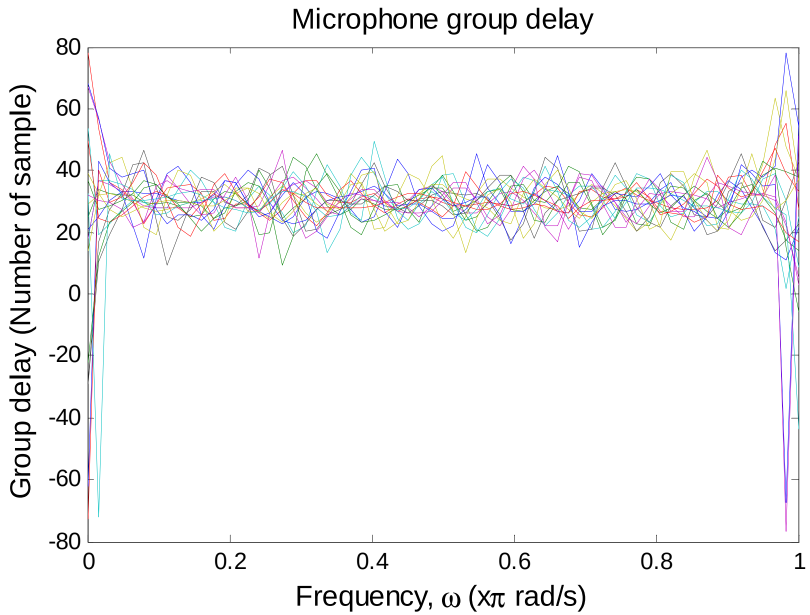
Parameters	Values
Steering range	From -36^o to 36^o
Spectral passband	From 200 to 3800 Hz
Gain deviation PDF	Rayleigh(σ=1)
Phase deviation PDF	Uniform(-π/2, π/2)
No. of rings, P	4
Ring radii	0.033, 0.089, 0.242, 0.657 m
No. of mic per ring, K	5
Order of Farrow structure, M-1	4
No. of filter tap, N	32

Model error description

- Mismatch between microphones: Each microphone response is modelled as 50-tap bandpass FIR filter. Then, the filter coefficients are perturbed by a uniform random variables, i.e.

$$\hat{w} \sim w + U\left(-0.1, 0.1\right)$$



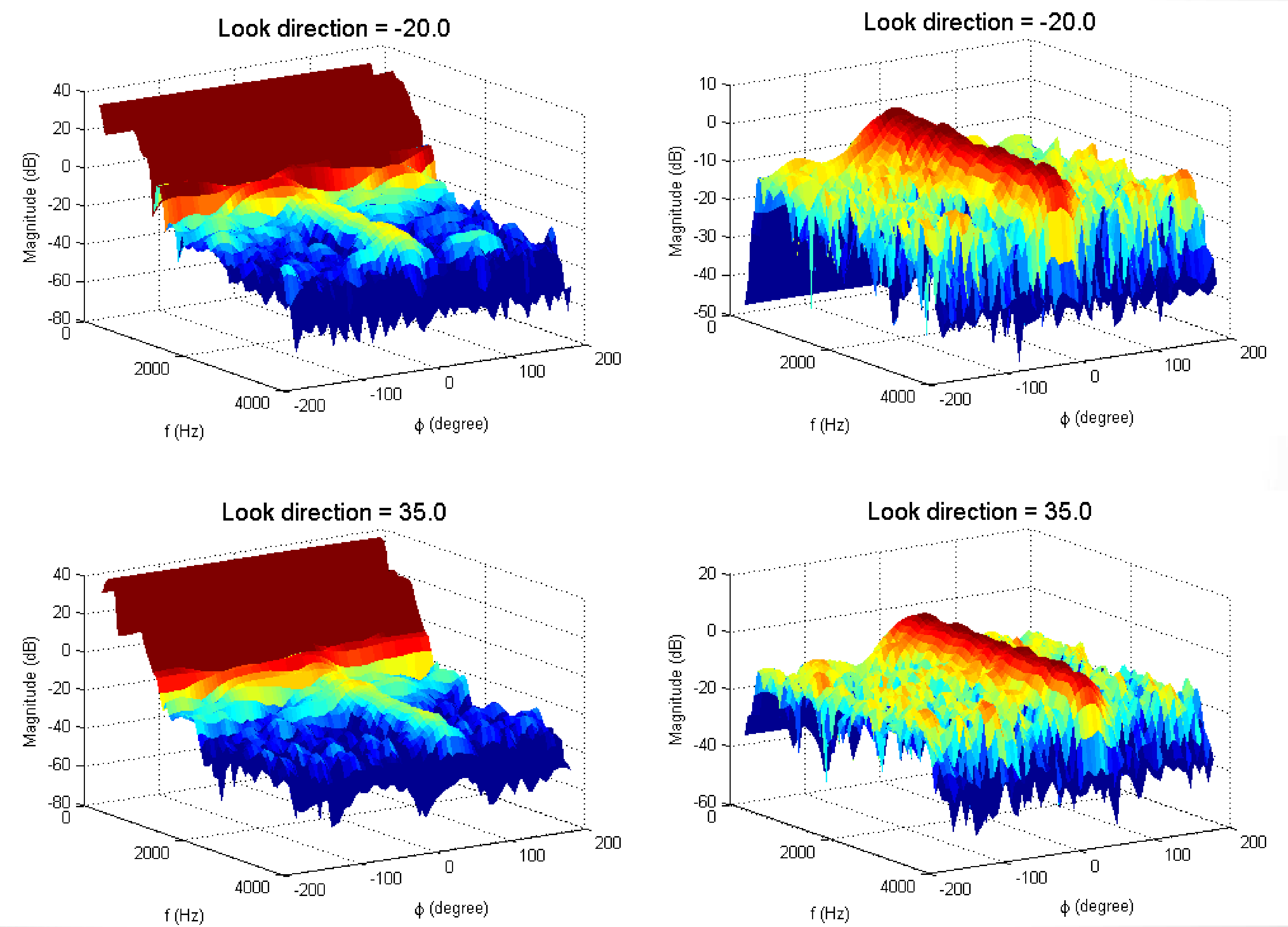


Model error description

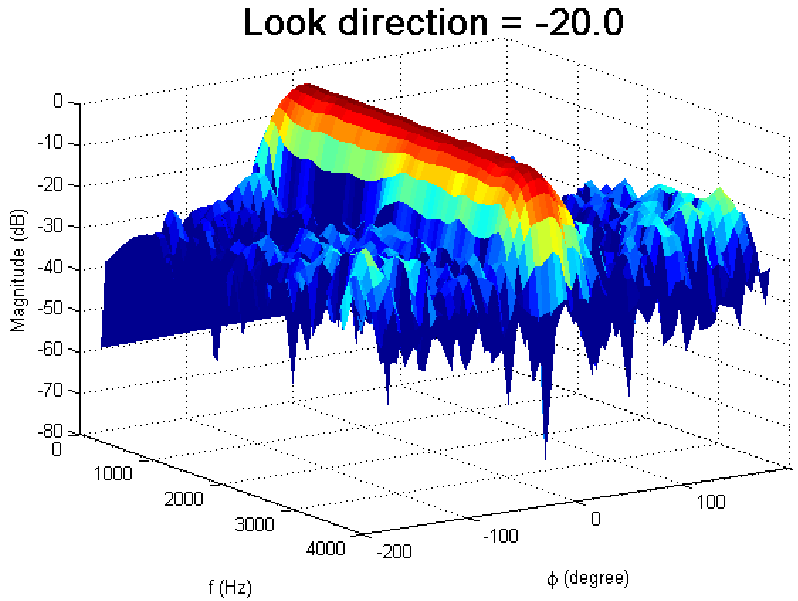
- Deviation from actual array geometry: Microphones' positions are perturbed with zero-mean Gaussian PDF with s.d. of 1cm

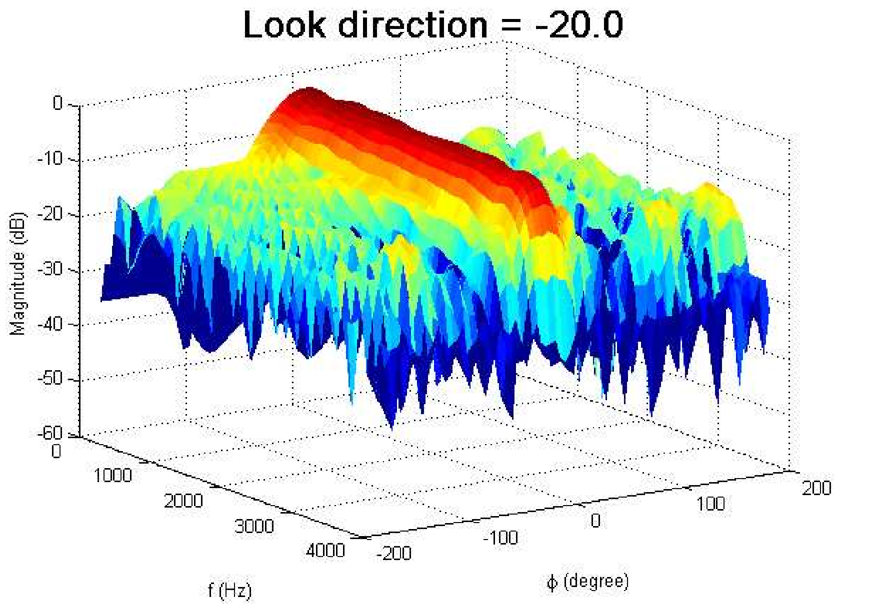
$$[\hat{x}_k, \hat{y}_k] \sim N\left([x_k, y_k], \begin{bmatrix} 0.01^2 & 0 \\ 0 & 0.01^2 \end{bmatrix} \right)$$

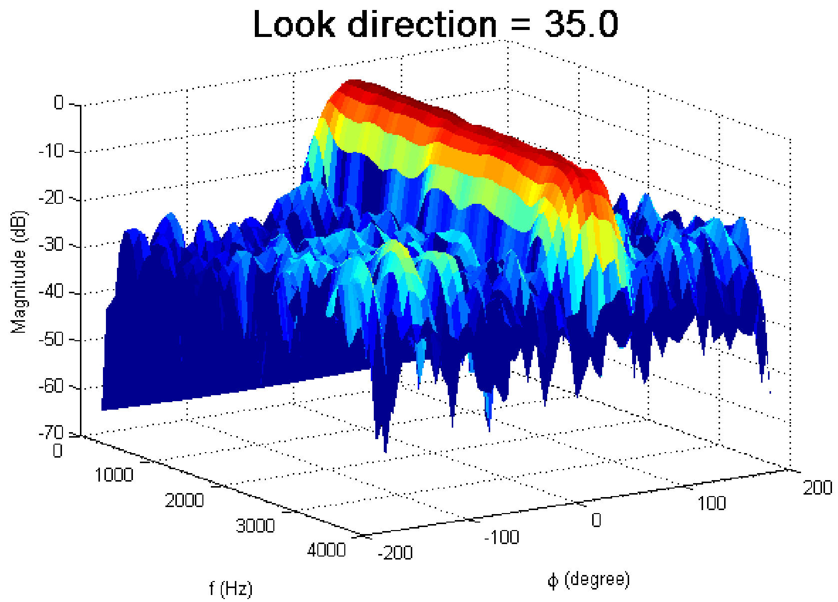
Results (With perturbation)

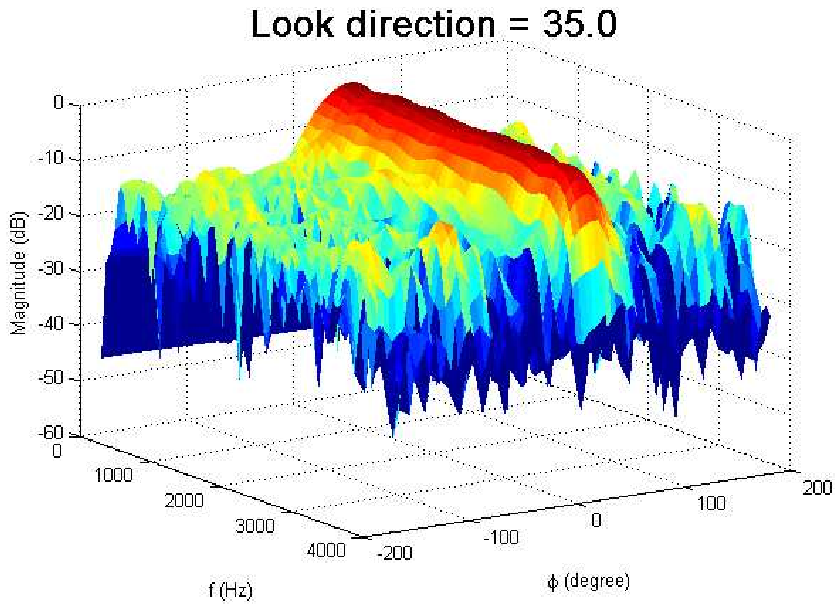


Results (No perturbation)









Conclusion

- We have proposed a robust SBBF design
- Steering capability is achieved by Farrow's filter structure
- Robust formulation using stochastic model and optimise for mean performance
- Design example shows:

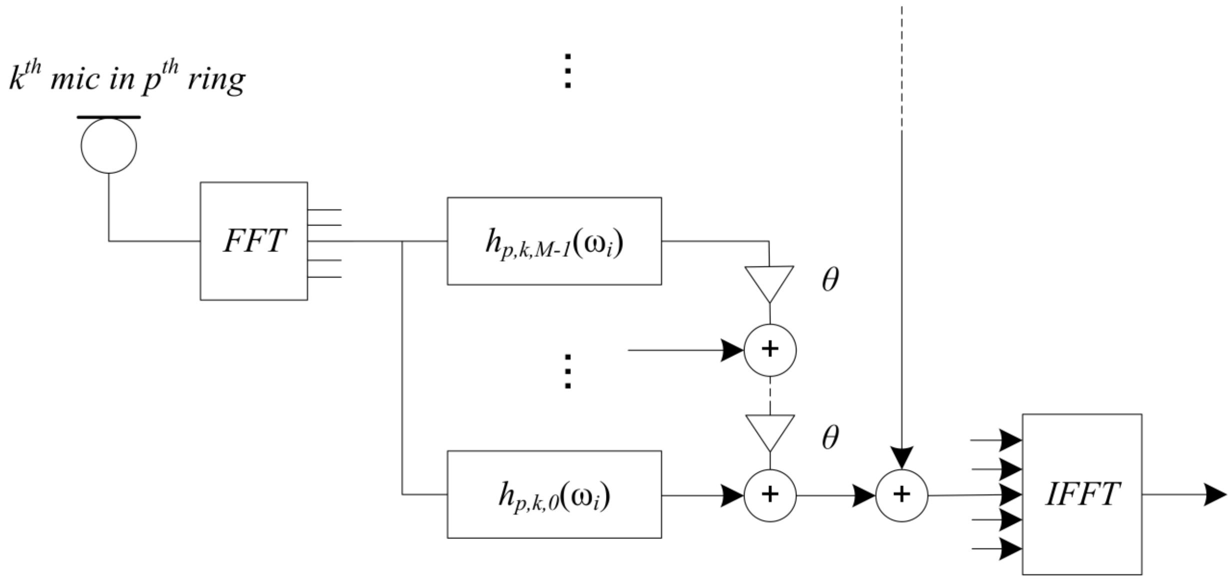
a) Steering ability (beampattern is maintained)

b) Frequency invariant response

c) Robustness against perturbation

Thank you

SBBF structure (FD)



- Beampattern:

$$\hat{G}(\psi,\omega,\phi)=\sum_{p}\sum_{k}\sum_{m}h_{k,m}(\omega)\hat{d}_{k}(\omega,\phi)\left(\psi/(2\psi_{\max})\right)^{m}$$
$$=\hat{\mathbf{a}}_{f}^{\mathrm{H}}(\psi,\omega,\phi)\mathbf{h}(\omega)$$

Robust design

- If we assume:
- a) Independence between microphone errors
- b) Independence between gain and phase errors
- c) All microphone error has similar model

- The mean error is given by

$$\bar{\mathbf{Q}}_\delta(\psi,\omega,\phi) = \left(\mu_\kappa^2\left[\left(\mu_\gamma^c\right)^2 + \left(\mu_\gamma^s\right)^2\right](\mathbf{1}_K - \mathbf{I}_K) + \sigma_\kappa^2\mathbf{I}_K\right)\otimes\mathbf{1}_{MN}$$

$$\bar{b}_\delta(\psi,\omega,\phi) = \mu_\kappa\left(\mu_\gamma^c + j\mu_\gamma^s\right)$$

- Variables $\mu_\kappa, \mu_\gamma^c, \mu_\gamma^s$ and σ_κ are found from the expected value of microphone errors $E\left\{\kappa_k(\omega,\phi)\exp\left(j\gamma_k(\omega,\phi)\right)\right\}$