Spatial Sound Reproduction System using Higher Order Loudspeakers

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Outline

Background

- Spatial sound reproduction systems aim to reproduce an arbitrary desired sound field within a region of space.
- The desired sound field may be generated using
  - The Kirchhoff-Helmholtz (K-H) integral (Wave Field Synthesis (WFS)), or
  - Cylindrical or spherical harmonic decompositions (higher order Ambisonics).

Issues with Spatial Sound Reproduction

- The accuracy of sound reproduction is governed by the wavelength (λ) and the size of the region over which reproduction is required.

For wave number $k = 2\pi f/c = 2\pi/\lambda$ and reproduction radius r the number of required loudspeakers (L) in the 2D case is given approximately by

$$L \approx 2kr + 1$$

- Thus, large numbers of loudspeakers are required for the reproduction of high frequencies over significant areas.
- E.g., reproduction over 0.1 m radius at 16 kHz requires 60 loudspeakers!.
- In the 3D case: $L \approx (kr + 1)^2$.

Issues with Spatial Sound Reproduction

- Loudspeakers produce a reverberant field which corrupts the sound field within the array.
- This reverberant field can be cancelled using calibration and pre-processing but such techniques require accurate measurement of acoustic transfer functions and significant computing power.
- If loudspeakers with omnidirectional and radial dipole directivities are used, it is possible to produce a sound field within the loudspeaker array and no exterior field, by using the K-H integral. (Exterior Field Cancellation).

Nyquist Frequency of the Array

- Nyquist frequency of the array is the corresponding frequency where transducers are a half wavelength apart.
- Exterior cancellation is possible below the Nyquist frequency of the array. At frequencies at or above Nyquist, the K-H approach fails and a nonzero exterior sound field is produced
- For 2D case with omnidirectional loudspeakers, $f^{(0)} = c(L-1)/4\pi r_L$.
- Literature (e.g., Vries, Ahrens etal, Poletti etal) shows that fixed-directivity speakers can reduce exterior field.

Summary of Issues with Spatial Sound Reproduction

- Large numbers of loudspeakers are required for the reproduction of high frequencies over significant areas.
- Loudspeakers produce a reverberant field (due to exterior field) which corrupts the sound field within the array.

Key Contributions from this Paper

- Use higher order variable directivity loudspeakers for sound reproduction. (2D case in this paper)
- Show that a circular array of Nth order loudspeakers produces a Nyquist frequency of N times that of a omnidirectional array.
- Hence, the reproduction area, or equivalently, the bandwidth of accurate reconstruction over a specified area, is increased by a factor N.
- Alternatively, the sound reproduction can be carried out using $1/N$th of the number of simple loudspeakers.

This paper is not about ...

We do not consider *how to design a Nth order variable directivity Loudspeaker*

Desired Interior Sound field

The desired sound field (2D case) in the interior of the loudspeaker array can be written as

$$p(r,\phi,k) = \sum_{n=-\infty}^{\infty} A_n(k)\, J_n(kr)\, e^{in\phi}$$

- $J_n(\cdot)$ is the nth cylindrical Bessel function, and $A_n(k)$ is the nth expansion coefficient.
- Due to the properties of the Bessel function, this expansion can be truncated to order $N \approx kr$ for a radius r and wavenumber k.
- With L loudspeakers, $L = 2N + 1$.

Desired Interior Sound field

The desired sound field (2D case)

$$p(r,\phi,k) = \sum_{m=-N}^{N} A_n(k)\, J_n(kr)\, e^{in\phi}$$

- We wish to reproduce the above field within a circular array of L loudspeakers positioned at (r_ℓ, ϕ_ℓ), $\ell = 1, \ldots, L$.

Zeroth order Sound reproduction System

In a simple zeroth order loudspeaker sound reproduction system, each loudspeaker is a monopole given by $H_0^{(1)}(k||\boldsymbol{r} - \boldsymbol{r}_\ell||)$ with the expansion

$$H_0^{(1)}(k||\boldsymbol{r} - \boldsymbol{r}_\ell||) = \begin{cases} \sum_{m=-\infty}^{\infty} J_m(kr) H_m^{(1)}(kr_\ell)\, e^{jm(\phi-\phi_\ell)} & \text{for } r < r_\ell \\ \sum_{m=-\infty}^{\infty} J_m(kr_\ell) H_m^{(1)}(kr)\, e^{jm(\phi-\phi_\ell)} & \text{for } r_\ell < r \end{cases}$$

where $H_m^{(1)}(\cdot)$ is the mth cylindrical Hankel function of the first kind.

Nth order Sound sources

Nth order Sound source can be represented as $H_n^{(1)}(k||\boldsymbol{r} - \boldsymbol{r}_\ell||)e^{jn\beta_\ell}$, where β_ℓ is the angle measured from the field point $\boldsymbol{r}$ to the source point vector $\boldsymbol{r}_\ell$.

$H_n^{(1)}(k||\boldsymbol{r} - \boldsymbol{r}_\ell||)e^{jn\beta_\ell}$ can be expressed:

$$= \begin{cases} \sum_{m=-\infty}^{\infty} J_m(kr)H_{m+n}^{(1)}(kr_\ell)\, e^{jm(\phi-\phi_\ell)} \text{ for } r < r_\ell \\ \sum_{m=-\infty}^{\infty} J_{m+n}(kr_\ell)H_m^{(1)}(kr)\, e^{jm(\phi-\phi_\ell)} \text{ for } r_\ell < r \end{cases}$$

Nth order Sound sources

- A combination of order $n = N$ and $n = -N$ sources has a far-field polar response which is a combination of $\cos N\phi$ and $\sin N\phi$ terms.
- A loudspeaker which produces a general Nth order source consists of a sum of source orders $n \in [-N, N]$ with weights $w_{n,\ell}$.
- An array of L general Nth order sources will produce a field

$$p(r, \phi, k) = \sum_{\ell=1}^{L} \sum_{n=-N}^{N} w_{n,\ell} H_n^{(1)}(k||\mathbf{r} - \mathbf{r}_\ell||) e^{jn\beta_\ell}$$

Solution via Mode Matching

By equating the desired interior field to the field created by L general Nth order sources, and mode matching:

Interior Mode Matching Equation

$$\sum_{n=-N}^{N} H^{(1)}_{m+n}(kr_L) \sum_{\ell=1}^{L} w_{n,\ell} e^{-jm\phi_\ell} = A_m, \;\; m \in [-M, M].$$

Assuming zero desired exterior field

Exterior Mode Matching Equation

$$\sum_{n=-N}^{N} J_{m+n}(kr_L) \sum_{\ell=1}^{L} w_{n,\ell} e^{-jm\phi_\ell} = 0, \;\; m \in [-M, M].$$

Solution via Mode Matching

- We can combine *interior* and *exterior* equation together to form a Matrix equation in the form of

$$\boldsymbol{H}\boldsymbol{w} = \boldsymbol{d}$$

  where $\boldsymbol{H}$ is $(4M+2) \times (2N+1)L$ matrix, $\boldsymbol{w}$ is a $(2N+1)L$ vector and $\boldsymbol{d}$ is a $2M+1$ vector of desired sound field coefficients.

- This matrix equation can be solved for higher order source weights when $(4M+2) \leq (2N+1)L$.

Solution via Mode Matching

- To be able to reproduce the interior field as well as to cancel the exterior field, we need $(4M + 2) \leq (2N + 1)L$.
- M is the number of modes supported in the interior field and N is the order of the each loudspeaker unit and L is the number of loudspeakers.
- Also $M \approx kr_M$ where r_M is the maximum reproduction radius.

Matlab Simulation

- Circular array containing $L = 15$ higher order loudspeakers.
- Array radius $r_L = 3$ m.
- Corresponds to Nyquist frequency of 126 Hz.
- Desired field is due to a line source located at a radius 6 m and an angle 36° degrees- half way between adjacent loudspeakers.

First order: Performance just below Nyquist

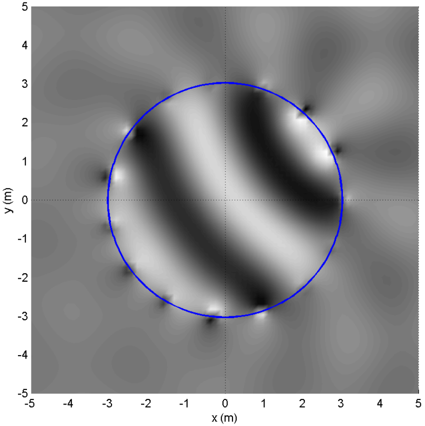

Figure: Sound field for N=1 loudspeakers at frequency 125 Hz

First order: Performance above Nyquist

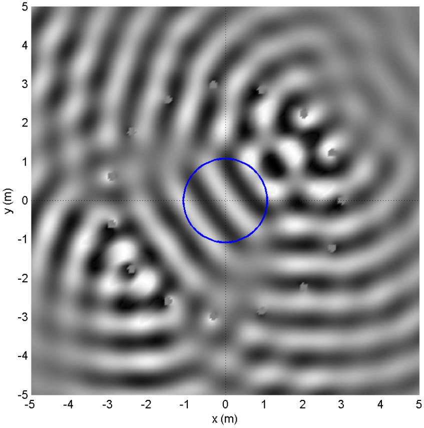

Figure: Sound field for N=1 loudspeakers at frequency 350 Hz

Reproduction Error

The Reproduction error is defined relative to the desired pressure at the origin

$$\epsilon(r,\phi,k) = \begin{cases} |\hat{p}(r,\phi,k) - p(r,\phi,k)|/|p(0,0,k)|, & r < r_L, \\ |\hat{p}(r,\phi,k)|/|p(0,0,k)|, & r > r_L. \end{cases}$$

First order: Reproduction Error

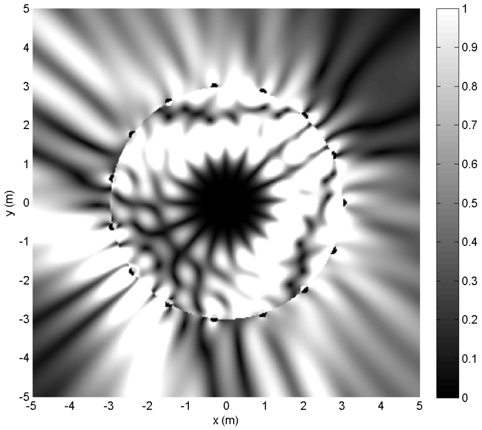

Figure: Reproduction error for N=1 array at frequency 350 Hz

Third order array

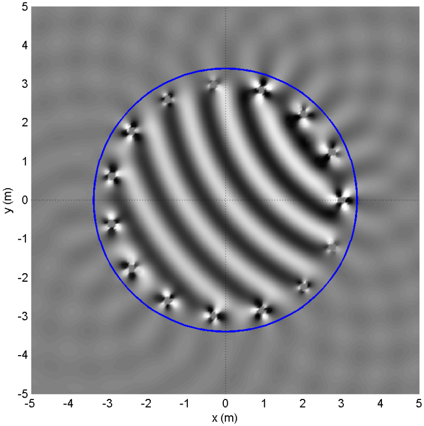

Figure: Sound field for N=3, frequency 350 Hz

Reproduction error: Third Order

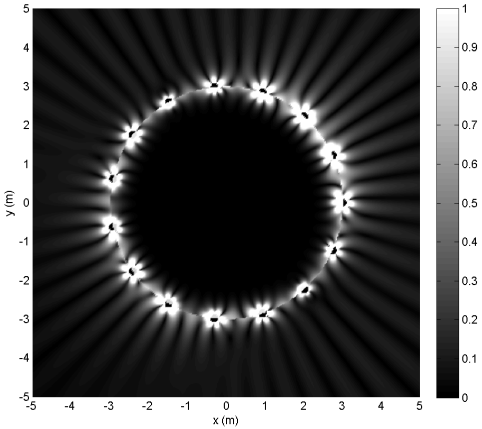

Figure: Reproduction error, N=3, frequency 350 Hz

6th Order Array @ 800 Hz

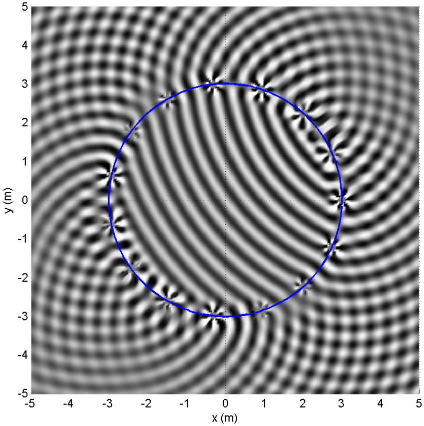

Figure: Sound field for N=6, frequency 800 Hz

Conclusions

- Considered the use of an array of higher order loudspeakers for sound field reproduction, in the 2D case.
- We have demonstrated by simulation that an Nth order array - capable of radiating polar responses up to and including $\cos N\phi$ and $\sin N\phi$ - is able to extend the reproduction region, or the frequency range, by a factor N, while significantly reducing the exterior sound field.
- This suggests that sound reproduction can be carried out using $1/N$th of the number of simple loudspeakers, if those loudspeakers are able to produce all responses up to Nth order.