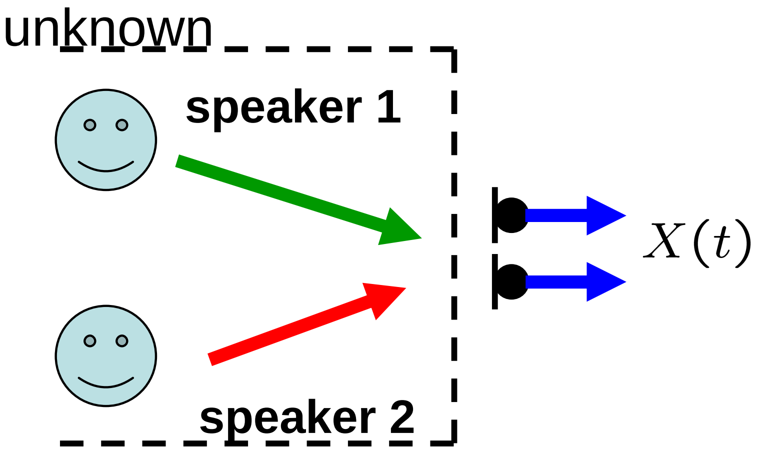
RESOLVING FD-BSS PERMUTATION FOR ARBITRARY ARRAY IN PRESENCE OF SPATIAL ALIASING

Jani Even, Norihiro Hagita
even@atr.jp

Introduction

Cocktail party (speech/speech)



- Blind signal separation (BSS)
- Frequency domain
- Permutation resolution
- DOA based approach
- Spatial aliasing

- Model for the spatial aliasing
- Sparse solution
- Permutation resolution

Outline

1. Preliminaries
   1. FD-BSS & Permutation
   2. Method based on DOA estimation
2. Proposed approach
   1. Spatial aliasing equation
   2. Finding the solution
   3. Permutation resolution
3. Simulation results

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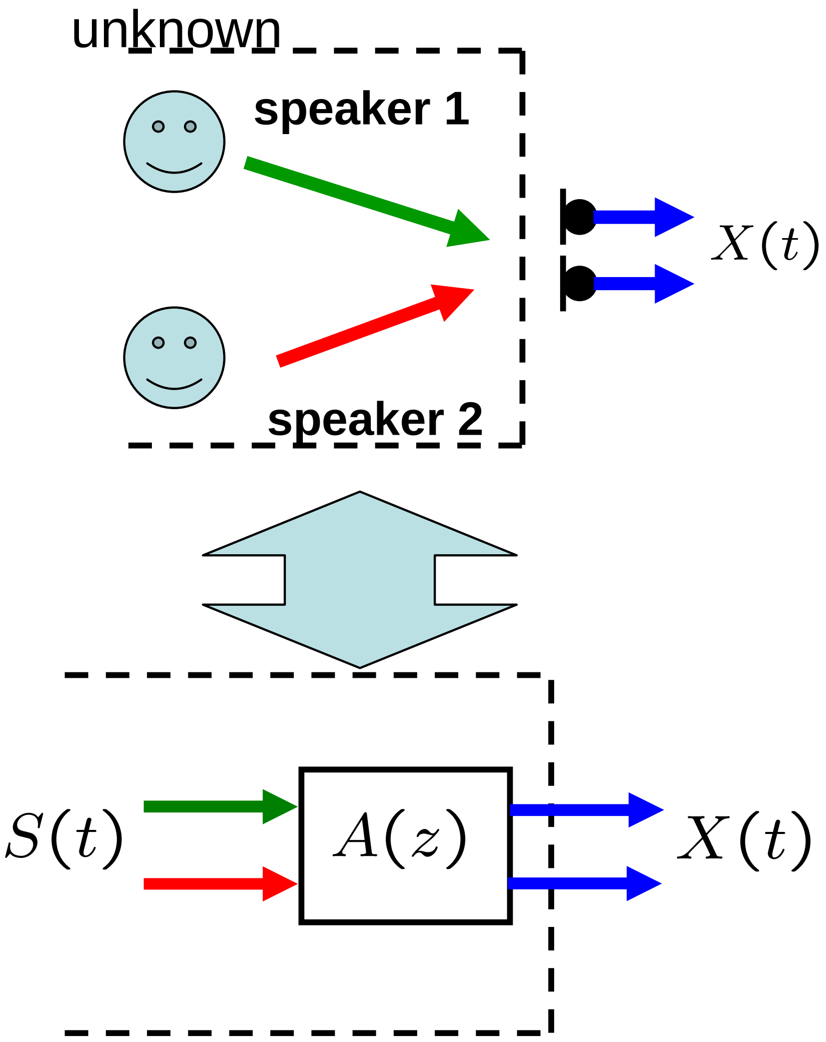
Frequency domain mixture

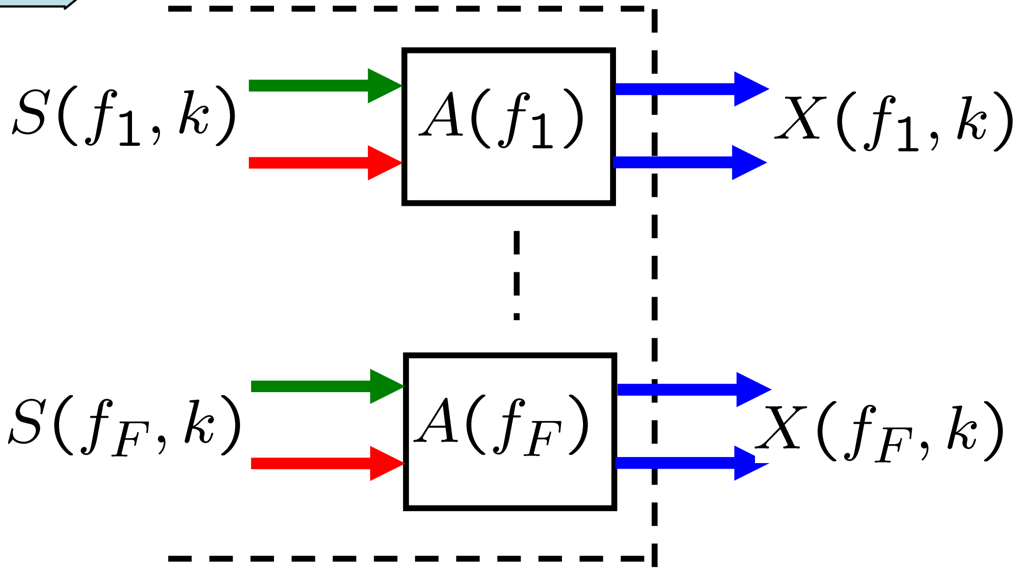
Time domain
Convolutive mixture



Unknown

STFT

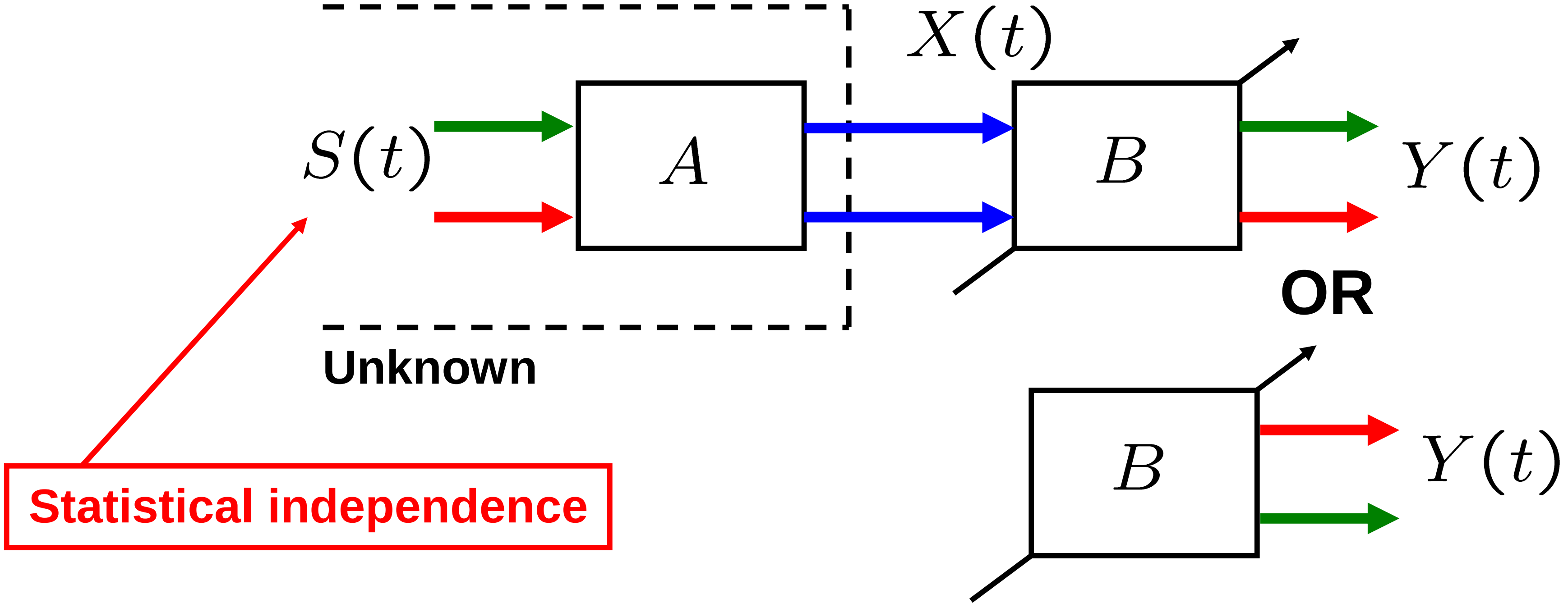
Frequency domain
instantaneous mixtures



Unknown

- **User 1**
- **User 2**
- **Mixtures**

Frequency domain BSS



(independent component analysis ICA)

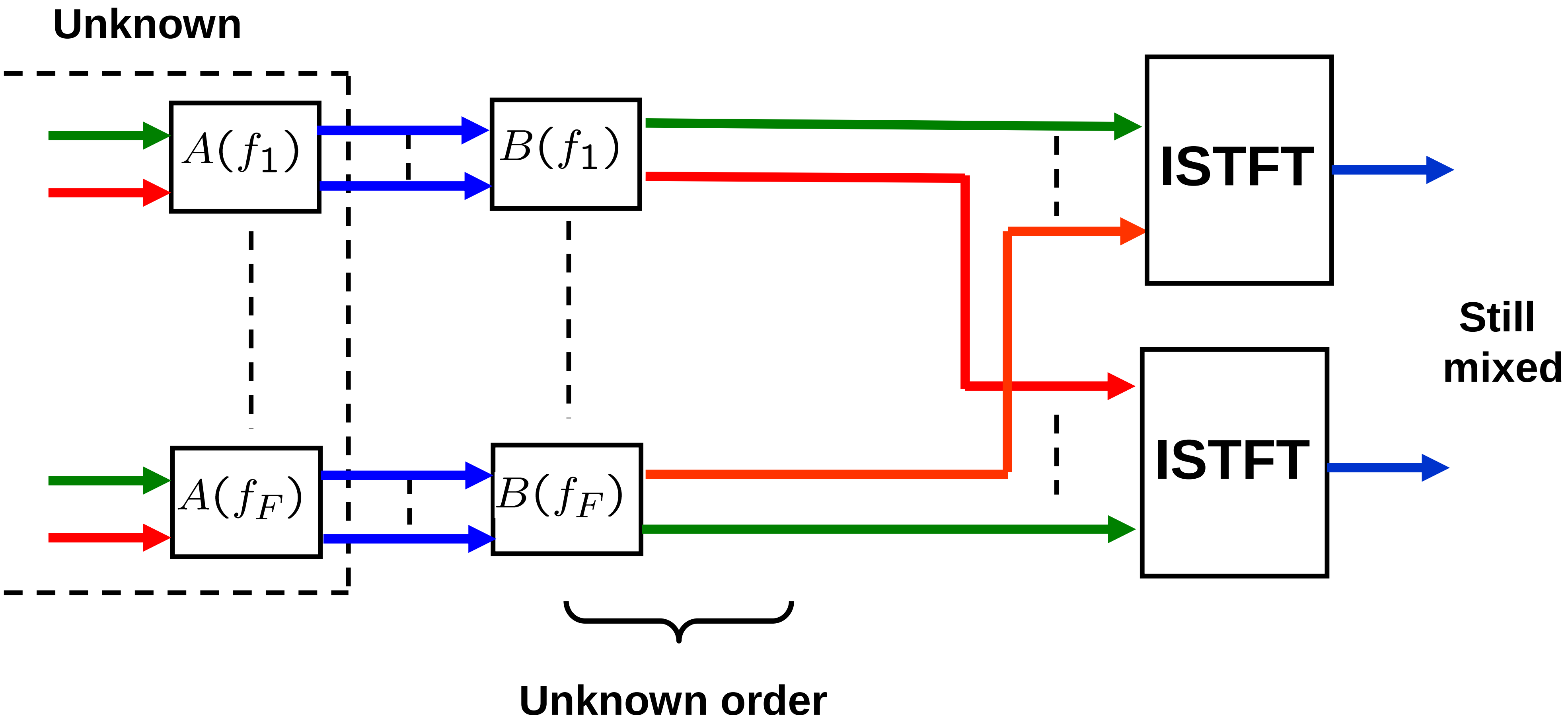
🟩 **Speech 1**
🟥 **Speech 2**
🟦 **Mixture**

$$Y(t) = P\Lambda S(t)$$

Λ **diagonal matrix**

P **permutation matrix**

Need for permutation resolution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$A(f_1)$

$A(f_F)$

$B(f_1)$

$B(f_F)$

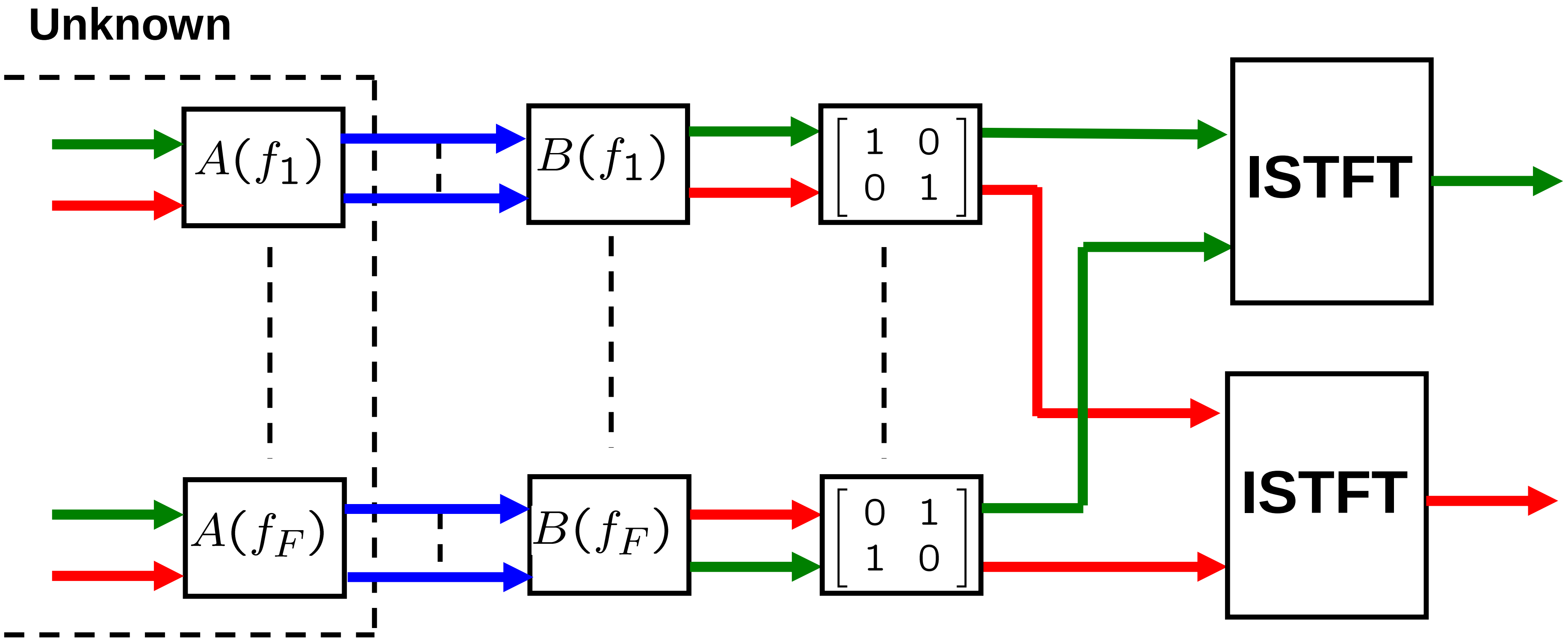
ISTFT

ISTFT

Still mixed

Unknown order

Set of permutation matrices



Permutation resolution ⟺ **Determine *F* permutation matrices**

Permutation resolution methods

See review paper [Pedersen, Larsen, Kjems, Parra 2007]

1. **Consistency of the spectrum of the recovered signals**
    1. **Amplitude modulation**
    2. **Cross correlation**
    3. **Source distribution**
    4. **…**
2. **Consistency of the filter coefficients**
    1. **Initialization**
    2. **Smooth spectrum**
    3. **Directivity pattern**
    4. **DOA**

Results in presence of spatial aliasing:

[Morgan, Ikram 2002]
[Sawada,Araki,Mukai,Makino, 2006]
[Nesta, Omologo,2010]

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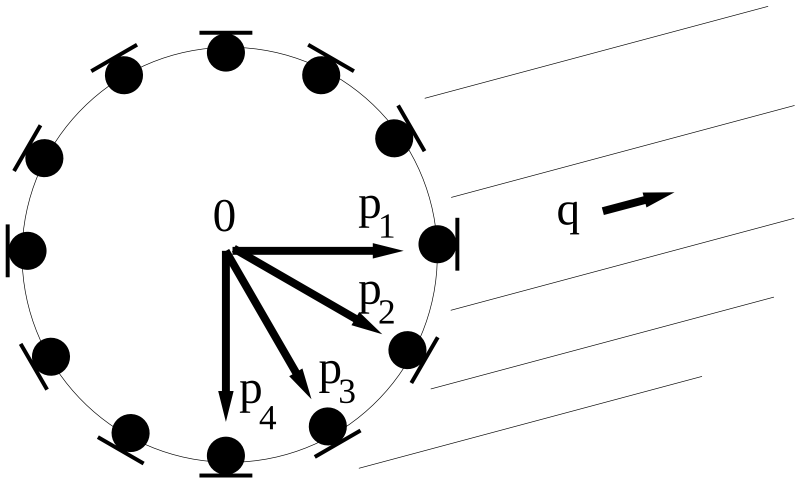
Steering vector model

Far field assumption



Attenuation and delay model

$$\mathbf{A}(f) \approx \begin{bmatrix} \frac{1}{\lambda_1} e^{-j2\pi \frac{f}{c}\mathbf{p}_1\cdot\mathbf{q}_1} \dots & \frac{1}{\lambda_n} e^{-j2\pi \frac{f}{c}\mathbf{p}_1\cdot\mathbf{q}_n} \\ \vdots & \vdots \\ \frac{1}{\lambda_1} e^{-j2\pi \frac{f}{c}\mathbf{p}_n\cdot\mathbf{q}_1} \dots & \frac{1}{\lambda_n} e^{-j2\pi \frac{f}{c}\mathbf{p}_n\cdot\mathbf{q}_n} \end{bmatrix}$$

$$\mathbf{q}_1 \text{ - - - - - - - - } \mathbf{q}_n$$

Absolute DOAs

Estimated mixing matrix approximation from BSS

$$\widehat{\mathbf{A}(f)} = \mathrm{B}(f)^{-1} \approx \begin{bmatrix} \frac{1}{\lambda_1} e^{-j2\pi \frac{f}{c}\mathbf{p}_1\cdot\mathbf{q}_{\sigma(f,1)}} \dots & \frac{1}{\lambda_n} e^{-j2\pi \frac{f}{c}\mathbf{p}_1\cdot\mathbf{q}_{\sigma(f,n)}} \\ \vdots & \vdots \\ \frac{1}{\lambda_1} e^{-j2\pi \frac{f}{c}\mathbf{p}_n\cdot\mathbf{q}_{\sigma(f,1)}} \dots & \frac{1}{\lambda_n} e^{-j2\pi \frac{f}{c}\mathbf{p}_n\cdot\mathbf{q}_{\sigma(f,n)}} \end{bmatrix}$$

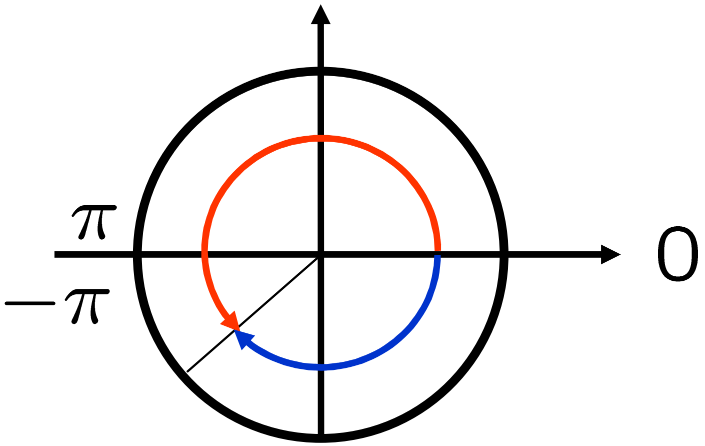
$$\mathbf{q}_{\sigma(f,1)} \text{ - - - - - - - - } \mathbf{q}_{\sigma(f,n)}$$

Permuted absolute DOAs

DOA estimation constraints

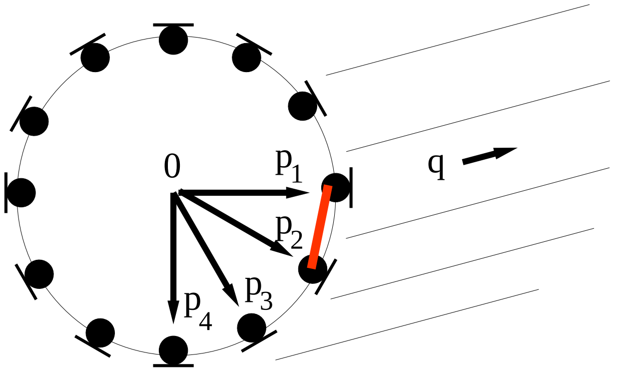
Ratio of coefficients

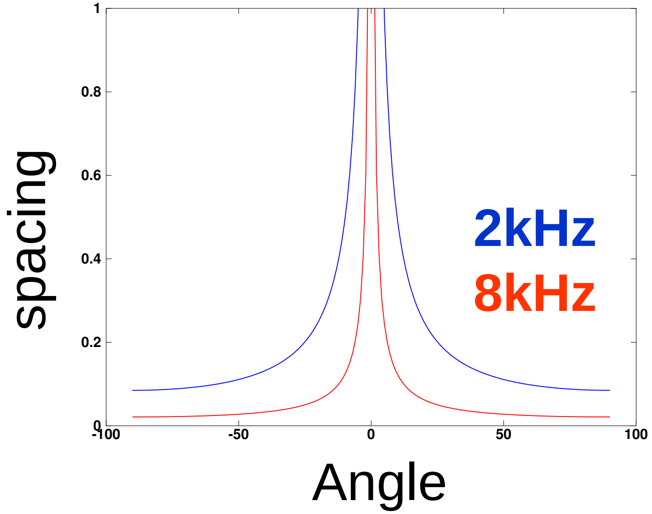
$$\arg\left(\frac{\widehat{\mathbf{A}}_{ij}(f)}{\widehat{\mathbf{A}}_{kj}(f)}\right) = -2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)},$$

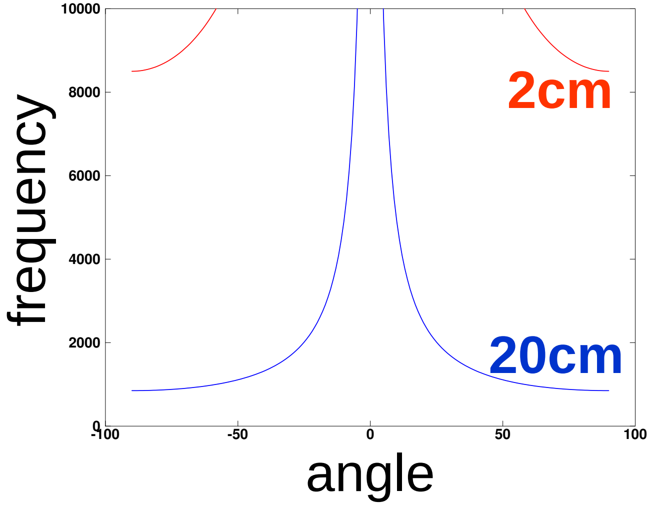


Limited to domain where arg is bijective

$$-\pi < 2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)} \le \pi$$







Use all sensors pairs

$$\arg\left(\frac{\widehat{\mathbf{A}}_{ij}(f)}{\widehat{\mathbf{A}}_{kj}(f)}\right) = -2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)},$$

Stack for all microphone pairs

$$L = \frac{n(n-1)}{2}$$

$$\mathbf{R}_j(f) = \begin{bmatrix} \arg\left(\frac{\widehat{\mathbf{A}}_{1j}(f)}{\widehat{\mathbf{A}}_{2j}(f)}\right) \\ \arg\left(\frac{\widehat{\mathbf{A}}_{2j}(f)}{\widehat{\mathbf{A}}_{3j}(f)}\right) \\ \vdots \\ \arg\left(\frac{\widehat{\mathbf{A}}_{n-1j}(f)}{\widehat{\mathbf{A}}_{nj}(f)}\right) \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1 - \mathbf{p}_2 \\ \mathbf{p}_2 - \mathbf{p}_3 \\ \vdots \\ \mathbf{p}_{n-1} - \mathbf{p}_n \end{bmatrix} \updownarrow L$$

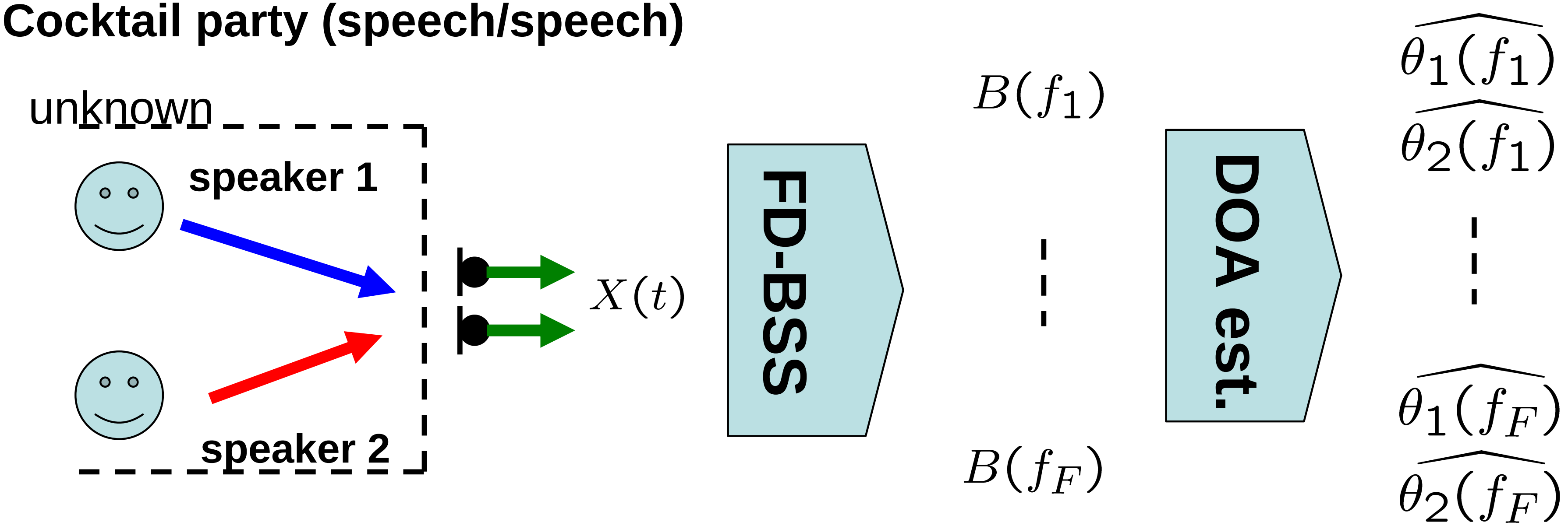
Least square solution

$$\mathbf{R}_j(f) = -2\pi\frac{f}{c}\mathbf{P}\boxed{\mathbf{q}_{\sigma(f,j)}} \Rightarrow \mathbf{q}_{\sigma(f,j)} = -\frac{c}{2\pi f}\mathbf{P}^{+}\mathbf{R}_j(f),$$

Unknown

Permutation alignment

Cocktail party (speech/speech)



unknown

speaker 1

speaker 2

$X(t)$

FD-BSS

$B(f_1)$

$B(f_F)$

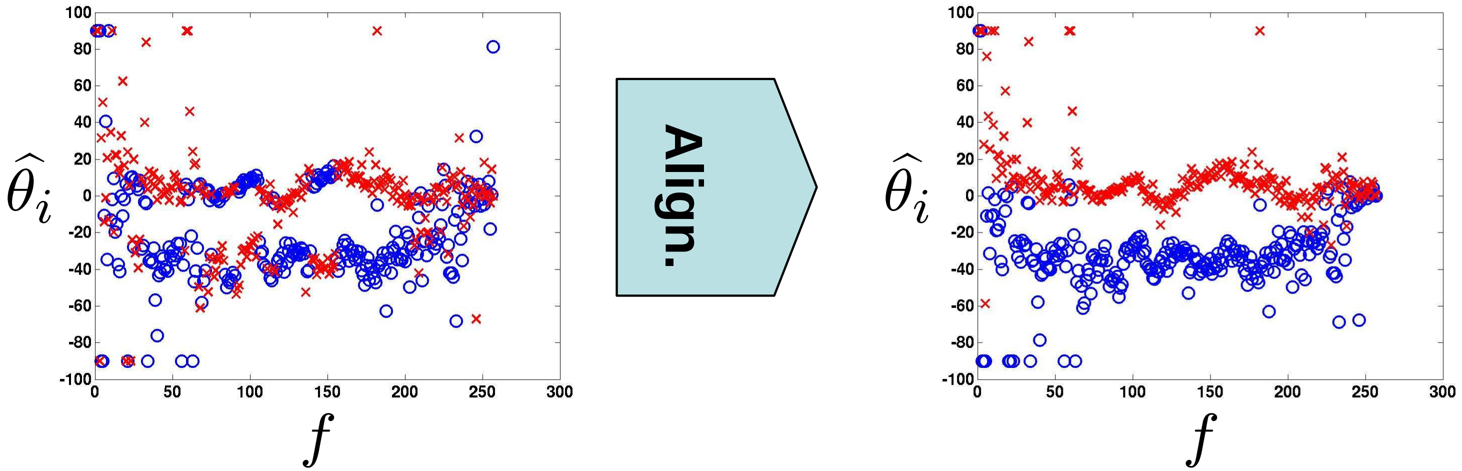
DOA est.

$\widehat{\theta_1(f_1)}$

$\widehat{\theta_2(f_1)}$

$\widehat{\theta_1(f_F)}$

$\widehat{\theta_2(f_F)}$



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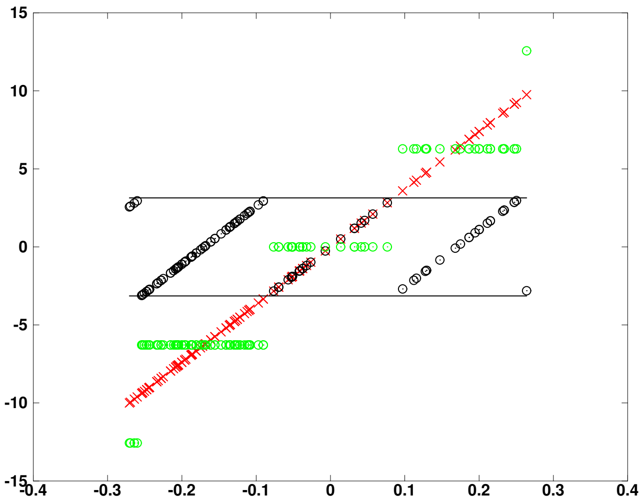
Including spatial aliasing

$$-\pi < 2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)} \leq \pi$$ **Not longer verified**

$$-\pi+\delta_{ijkf}2\pi < 2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)} \leq \pi+\delta_{ijkf}2\pi.$$

δ_{ijkf} **integers**

$$f = 2000\text{Hz}$$



$(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)}$

🟥 $2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)}$

🟩 $\delta_{ijkf}2\pi$

⬛ $2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)} - \delta_{ijkf}2\pi$

Use all microphone pairs

$$\arg\left(\frac{\widehat{\mathbf{A}}_{ij}(f)}{\widehat{\mathbf{A}}_{kj}(f)}\right) = -2\pi\frac{f}{c}(\mathbf{p}_i - \mathbf{p}_k)\cdot\mathbf{q}_{\sigma(f,j)} + 2\pi\delta_{ijkf}.$$

Stack column ratios, sensor differences and aliasing compensations

$$\mathbf{R}_j(f) = \begin{bmatrix} \arg\left(\frac{\widehat{\mathbf{A}}_{1j}(f)}{\widehat{\mathbf{A}}_{2j}(f)}\right) \\ \arg\left(\frac{\widehat{\mathbf{A}}_{2j}(f)}{\widehat{\mathbf{A}}_{3j}(f)}\right) \\ \vdots \\ \arg\left(\frac{\widehat{\mathbf{A}}_{n-1j}(f)}{\widehat{\mathbf{A}}_{nj}(f)}\right) \end{bmatrix} \quad \mathbf{P} = \begin{bmatrix} \mathbf{p}_1 - \mathbf{p}_2 \\ \mathbf{p}_2 - \mathbf{p}_3 \\ \vdots \\ \mathbf{p}_{n-1} - \mathbf{p}_n \end{bmatrix} \qquad \mathbf{\Delta}_j(f) = \begin{bmatrix} \delta_{12jf} \\ \delta_{23jf} \\ \vdots \\ \delta_{nn-1jf} \end{bmatrix}$$

$$\mathbf{R}_j(f) = -2\pi\frac{f}{c}\mathbf{P}\boxed{\mathbf{q}_{\sigma(f,j)}} + 2\pi\boxed{\mathbf{\Delta}_j(f)}$$

Unknown **Unknown**

Equation verified by $\Delta_j(f)$

$$\text{(I)} \quad \mathbf{R}_j(f) = -2\pi\frac{f}{c}\mathbf{P}\mathbf{q}_{\sigma(f,j)} + 2\pi\boldsymbol{\Delta}_j(f)$$

$$\text{(II)} \leftarrow \mathbf{P}\mathbf{P}^{+} \times \text{(I)}$$

left pseudo-inverse

$$\mathbf{P}^{+}\mathbf{P} = \mathbf{I}$$

$$\begin{aligned}\text{(II)} \quad \mathbf{P}\mathbf{P}^{+}\mathbf{R}_j(f) &= -2\pi\frac{f}{c}\mathbf{P}\mathbf{P}^{+}\mathbf{P}\mathbf{q}_{\sigma(f,j)} + 2\pi\mathbf{P}\mathbf{P}^{+}\boldsymbol{\Delta}_j(f)\\ &= -2\pi\frac{f}{c}\mathbf{P}\mathbf{q}_{\sigma(f,j)} + 2\pi\mathbf{P}\mathbf{P}^{+}\boldsymbol{\Delta}_j(f)\end{aligned}$$

$$\text{(II)} - \text{(I)}$$

$$\left(\mathbf{P}\mathbf{P}^{+} - \mathbf{I}\right)\mathbf{R}_j(f) = 2\pi\left(\mathbf{P}\mathbf{P}^{+} - \mathbf{I}\right)\boldsymbol{\Delta}_j(f).$$

$$\left\{\begin{aligned}&\mathbf{D}_j(f) = \frac{1}{2\pi}\left(\mathbf{P}\mathbf{P}^{+} - \mathbf{I}\right)\mathbf{R}_j(f)\\ &\mathbf{C} = \mathbf{P}\mathbf{P}^{+} - \mathbf{I}\\ &\mathbf{X} = \boldsymbol{\Delta}_j(f).\end{aligned}\right.$$

$$\mathbf{C}\mathbf{X} = \mathbf{D}_j(f)$$

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Solution of the equation

$$\mathrm{CX} = \mathrm{D}_j(f)$$

$$\mathbf{C} = \mathbf{P}\mathbf{P}^{+} - \mathbf{I}$$

$$L \times L$$

$$\mathrm{rank}(C) = L - \mathrm{rank}(P)$$

$$\left(\begin{array}{r} \mathbf{v} \in \ker(\mathbf{C}) \rightarrow \mathbf{v} = \mathbf{P}\mathbf{u} \\ \mathbf{u} = \mathbf{P}^{+}\mathbf{v} \end{array} \right)$$

$\mathrm{rank}(P)$ **Depends of array configuration**

➡ **Infinite number of real solutions**

Real solution with minimal norm $\mathrm{X} = \mathrm{C}^{+}\mathrm{D}_j(f)$

Finding integer solution $\mathrm{X} = \Delta_j(f)$ **?**

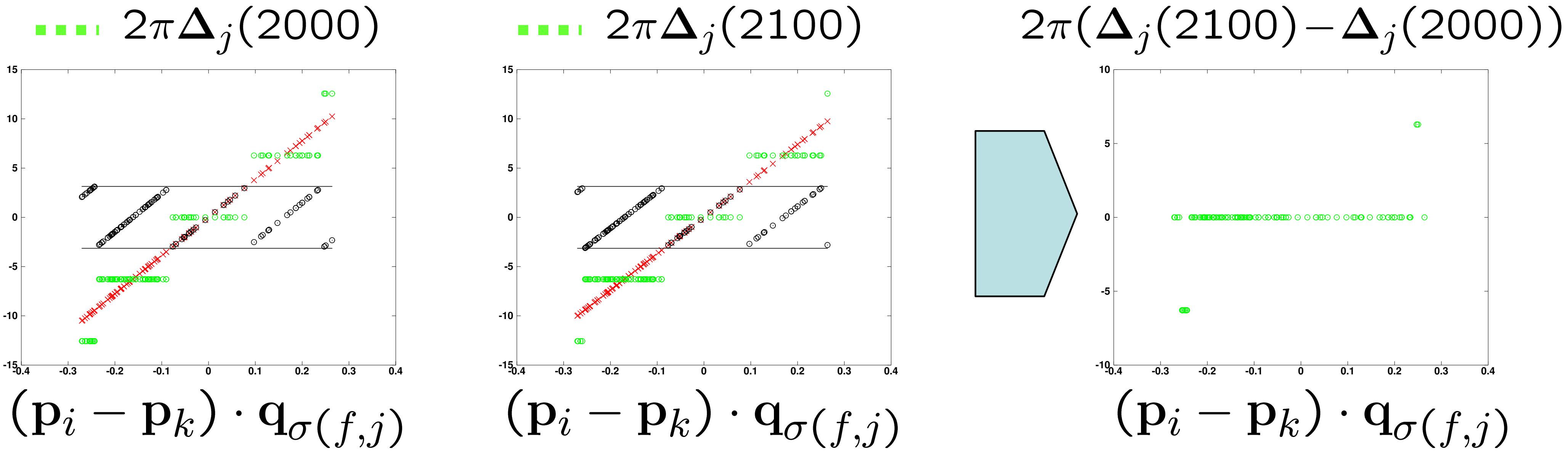
Sparse solution

$\mathbf{\Delta}_j(f)$ **null entries for row corresponding to microphone pair without aliasing**

"Good" initial guess $\widehat{\mathbf{\Delta}}_j(f)$

$\mathbf{\Delta}_j(f) - \widehat{\mathbf{\Delta}}_j(f)$ **sparser than** $\mathbf{\Delta}_j(f)$

Solution of $\mathbf{CX} = \mathbf{D}_j(f) - \mathbf{C}\widehat{\mathbf{\Delta}}_j(f)$



Find the solution

Use previous bin as initial guess $\widehat{\mathbf{\Delta}}_j(f) = \mathbf{\Delta}_j(f-1)$

Start from lower frequencies

$$\begin{cases} \mathbf{X} = \mathbf{C}^{+}\left(\mathbf{D}_j(f) - \mathbf{C}\mathbf{\Delta}_j(f-1)\right) + \mathbf{\Delta}_j(f-1) \\ \mathbf{\Delta}_j(f) = \lceil \mathbf{X} \rfloor \end{cases}$$

Goal: having $\mathbf{X}$ **close to** $\mathbf{\Delta}_j(f)$

Residual $\|\mathbf{C}\left(\mathbf{\Delta}_j(f) - \mathbf{D}_j(f)\right)\|$

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Permutation resolution

$\widehat{\mathbf{A}}(f-1)$

permutation
$k \rightleftharpoons j$

$\widehat{\mathbf{A}}(f)$

Equation in bin f **for row** k $\mathbf{CX} = \mathbf{D}_k(f) - \mathbf{C}\widehat{\mathbf{\Delta}}_j(f-1).$

$\widehat{\mathbf{\Delta}}_j(f-1) \neq \widehat{\mathbf{\Delta}}_k(f-1)$ **Close integer solution not found**

Residual $\left\|\mathbf{C}\left(\mathbf{\Delta}_j(f) - \mathbf{D}_j(f)\right)\right\|$ **large**

- **Compare to a threshold (hard to fix threshold with noise)**
- **Compare residual for different rows (more robust)**

$\widehat{\mathbf{\Delta}}_j(f-1) \approx \widehat{\mathbf{\Delta}}_k(f-1)$ **Close integer solution found**

Residual $\left\|\mathbf{C}\left(\mathbf{\Delta}_j(f) - \mathbf{D}_j(f)\right)\right\|$ **small**

Need to estimate $\mathbf{q}_{\sigma(f,j)} = -\frac{1}{2\pi f}\mathbf{P}^{+}(\mathbf{R}_j(f) + 2\pi\widehat{\mathbf{\Delta}}_j(f)).$

Post processing

Consider all frequency bins with all rows having small residuals

↓

Estimate the absolute DOAs using these frequency bins

↓

Average the DOAs along these frequency bins

↓

For all frequency bins and all rows,
compute the distances between the DOA and these averaged absolute DOAs

↓

cluster the rows according to these distances

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Simulation model

16 microphones uniform circular array (diam. 31cm, minimum spacing 3.12cm)

16kHz sampling frequency and 512 point fft

Model for the estimated mixing matrix $\widehat{\mathbf{A}}_{ij}(f) = \frac{1}{\lambda_1} e^{-j2\pi \frac{f}{c} \mathbf{p}_i \cdot \tilde{\mathbf{q}}_j} + \epsilon,$

$$\tilde{\mathbf{q}}_j = \begin{bmatrix} \cos(\theta)\sin(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\phi) \end{bmatrix}$$

Angle errors

$\{\theta + \epsilon_2, \phi + \epsilon_3\}$

$\{\epsilon_2, \epsilon_3\}$ **Uniform in** $[-\gamma, \gamma]$

Additive noise ϵ **Zero mean Gaussian with variance** β

Randomly permutation of two columns for a percentage *d* of the frequency bins

Measure of performance

r Percentage of frequency bins with adequate permutation

e Mean square error of the absolute DOA (in dB)

Experiment 1 Influence of $\epsilon, \ \gamma, \ d$

$$\tilde{\mathrm{q}}_1 \ \{40, 30\}$$

$$\tilde{\mathrm{q}}_2 \ \{-10, 25\}$$

Average results on 100 independent runs

Experiment 2 Influence of angle between $\tilde{\mathrm{q}}_1, \ \tilde{\mathrm{q}}_1$

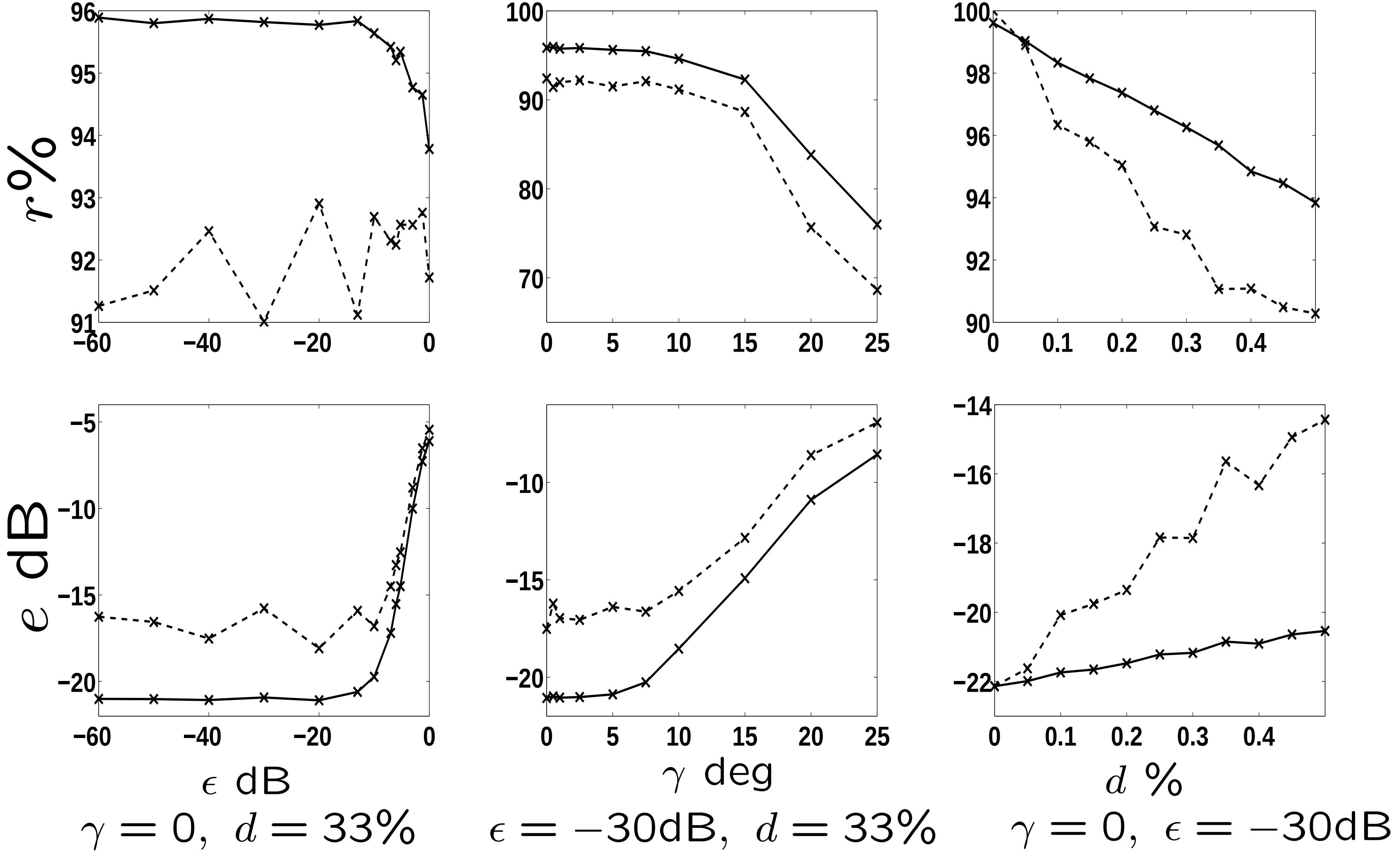
$$\epsilon = -30\mathrm{dB}, \ \gamma = 0, \ d = 33\%$$

Average results on 100 independent runs

Results 1

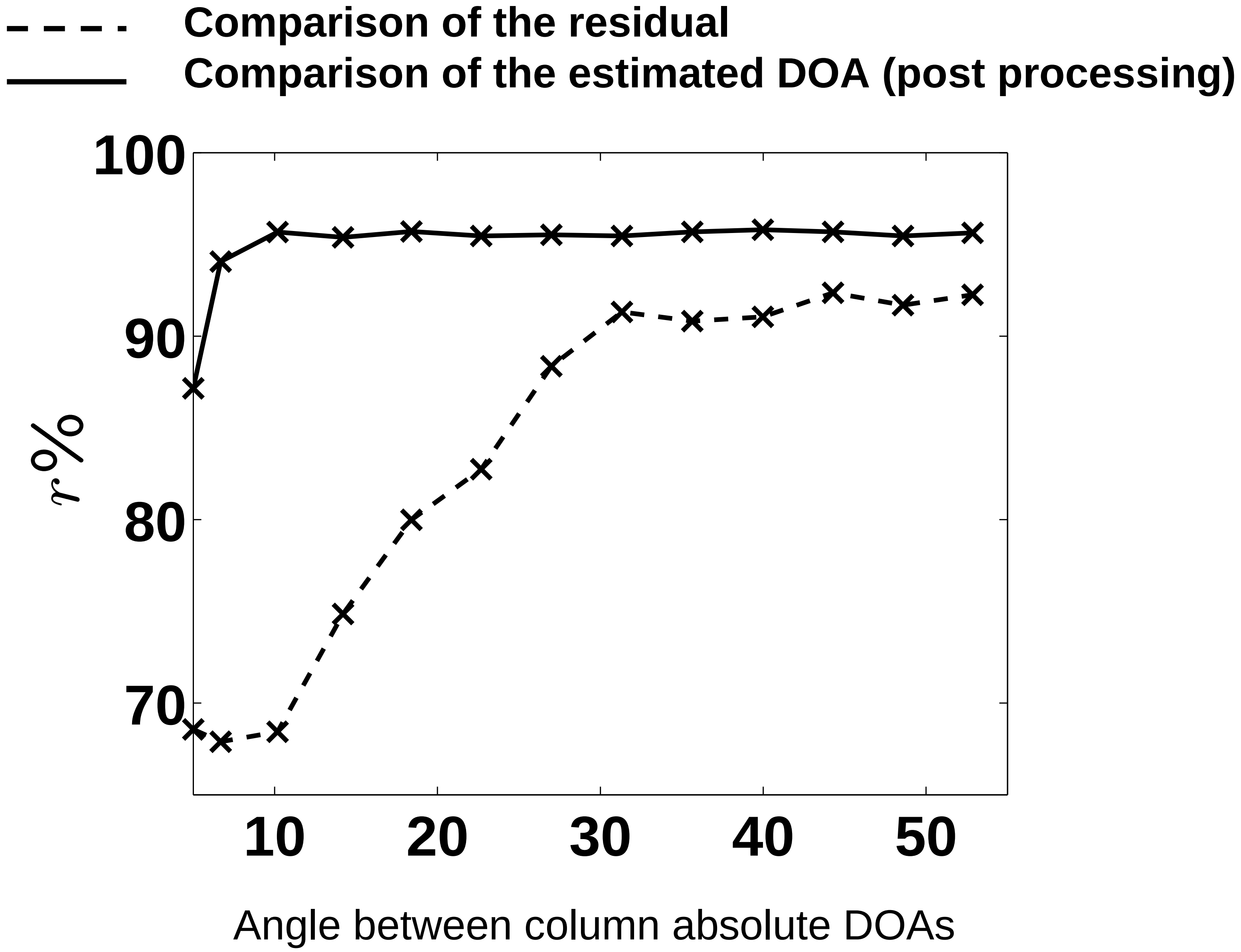
- - - - Comparison of the residual

—— Comparison of the estimated DOA (post processing)



$\gamma = 0,\ d = 33\%$ $\epsilon = -30\text{dB},\ d = 33\%$ $\gamma = 0,\ \epsilon = -30\text{dB}$

Results 2



Comparison of the residual
Comparison of the estimated DOA (post processing)
100
90
80
70
r%
10
20
30
40
50
Angle between column absolute DOAs

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Conclusion

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- Permutation resolution
- DOA based approach
- Spatial aliasing

- Model for the spatial aliasing
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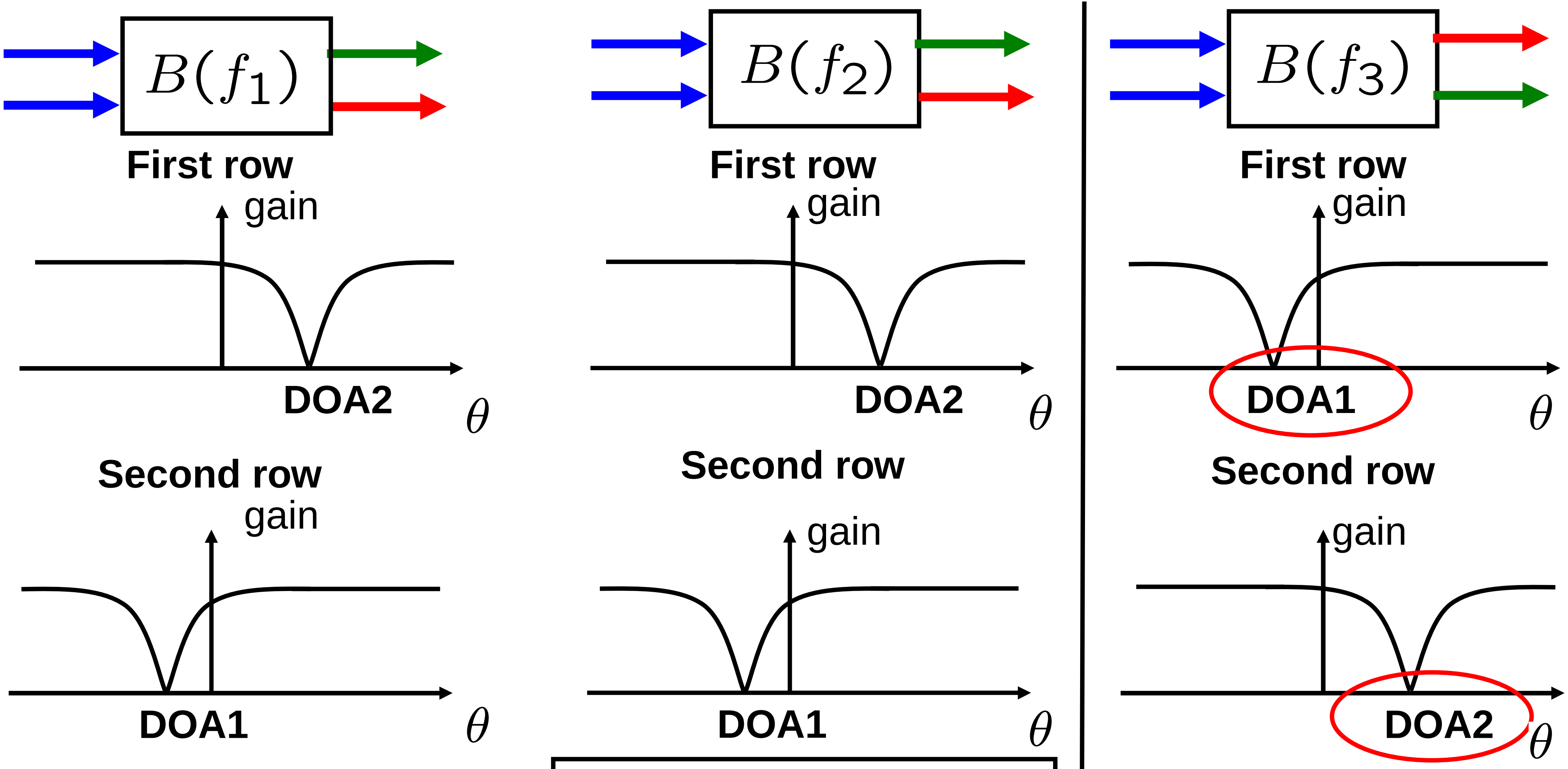
- Better resolution of the equation
- Apply to real data

Use of directivity pattern



Efficient suppression of Point sources

- 🟩 → **Speech 1 DOA1**
- 🟥 → **Speech 2 DOA 2**
- 🟦 → **Mixture**

Permuted