Transceiver Optimization for Multi-user Multi-antenna Two-way Relay Channels

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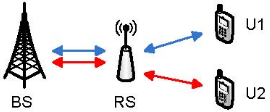
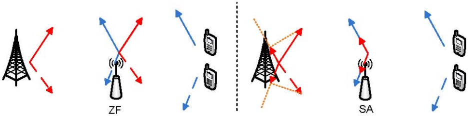
Two-way Cellular Systems



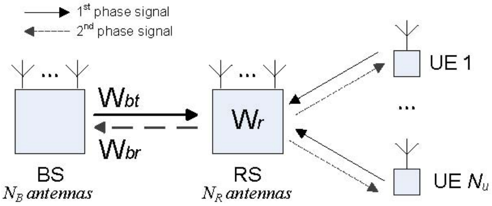
- Different users interfere with each other
- Exploit multi-antenna techniques at the RS and BS to combat the inter-user interference (IUI)
- AF protocol is considered

Existing IUI-free Schemes



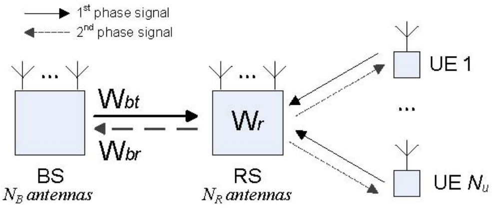
- For a N-user system:
  - Zero-forcing (ZF): $\geq 2N$ antennas at the RS.
  - Signal alignment (SA): $\geq N$ antennas at the RS. [Toh,2009, Ding,2011, Lee,2010]
  - Balanced Scheme: $\geq N$ antennas at the RS. Higher bidirectional sum rate than those of ZF and SA. [Sun,2010]
- Performance Benchmark?

System Model



- $\mathbf{H}_{br} \in \mathbb{C}^{N_R \times N_B}$ is the channel matrix from BS to RS
- $\mathbf{h}_{ir} \in \mathbb{C}^{N_R \times 1}$ is the channel vector from the ith user to RS
- Channels in two phases are assumed reciprocal

IUI-free Constraints



- BS receives (N_U uplink signals, N_U downlink signals);
  each user receives (N_U uplink signals, N_U downlink signals).
- Interference free constraints:

$$\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{H}_{br} \mathbf{w}_{btj} = 0, \ i \neq j$$

Sum Rate Maximization under IUI-free Constraints

- Sum rate:

$$R_U = \frac{1}{2}\sum_{i=1}^{N_u} \log_2\left(1 + \frac{P_R P_U |\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{ir}|^2}{N_0 P_R \|\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r\|^2 + N_0 \|\mathbf{w}_{bri}^T\|^2}\right)$$

$$R_D = \frac{1}{2}\sum_{i=1}^{N_u} \log_2\left(1 + \frac{P_R P_B |\mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{H}_{br} \mathbf{w}_{bti}|^2}{N_0 P_R \|\mathbf{h}_{ir}^T \mathbf{W}_r\|^2 + N_0}\right)$$

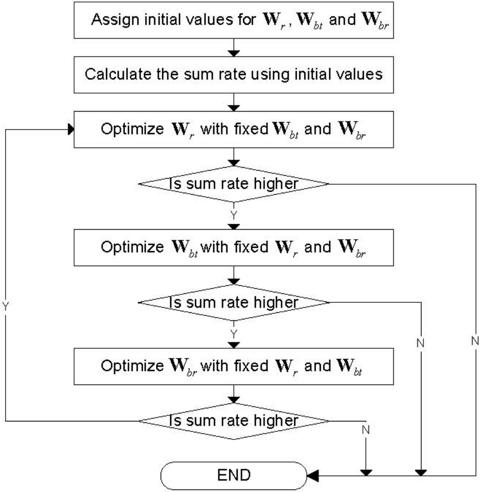
$$R_S = R_U + R_D$$

- Optimization problem:

$$\max_{\mathbf{W}_r, \mathbf{W}_{bt}, \mathbf{W}_{br}} R_S$$

$$\text{s.t.} \quad \text{IUI-free constraints}, \ \|\mathbf{W}_{bt}\|^2 \leq 1, \ \|\mathbf{W}_r \mathbf{y}_r\|^2 \leq 1.$$

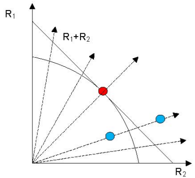
Procedure of Alternating Optimization



Optimization of $\mathbf{W}_r$ (1)

$$\max_{\mathbf{W}_r} \quad R_S$$
$$\text{s.t.} \quad \text{IUI-free constraints}, \ \|\mathbf{W}_r\mathbf{y}_r\|^2 \leq 1.$$

- Non-convex sum rate max problem $\rightarrow$ rate tuple [Mohseni,2006]



- Define $\boldsymbol{\beta} = \{\beta_1, \cdots, \beta_{2N_U}\}$, $\beta_i \geq 0$, $\sum \beta_i = 1$.
- The above problem =

$$\max_{\mathbf{W}_r} \quad R_S$$
$$\text{s.t.} \quad \text{IUI-free constraints}, \ \|\mathbf{W}_r\mathbf{y}_r\|^2 \leq 1,$$
$$R_{ui} \geq \beta_i R_S, \ R_{di} \geq \beta_{i+N_U} R_S$$

$+$ searching the optimal $\boldsymbol{\beta}$.

Optimization of $\mathbf{W}_r$ (2)

- Using the analogous approach in [Zhang,2009],

$$\begin{aligned} \max_{\mathbf{W}_r} \quad & R_S \\ \text{s.t.} \quad & \text{IUI-free constraints}, \ \|\mathbf{W}_r\mathbf{y}_r\|^2 \le 1, \\ & R_{ui} \ge \beta_i R_S, \ R_{di} \ge \beta_{i+N_U} R_S \end{aligned} \qquad \begin{aligned} \min_{\mathbf{W}_r} \quad & \|\mathbf{W}_r\mathbf{y}_r\|^2 \\ \text{s.t.} \quad & \text{IUI-free constraints}, \\ & R_{ui} \ge \beta_i R_S, \ R_{di} \ge \beta_{i+N_U} R_S \end{aligned}$$

- Let $x = \text{vec}(\mathbf{W}_r)$,

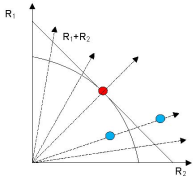
$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{x}^H \mathbf{K}_P \mathbf{x} \\ \text{s.t.} \quad & \mathbf{K}_O \mathbf{x} = \mathbf{0}, \ \mathbf{x}^H \mathbf{K}_i \mathbf{x} \ge c_i. \end{aligned}$$

- QCQP problem $\rightarrow$ a SDP problem with rank-1 constraint $\rightarrow$ SDR algorithm [Luo,2006].

Optimization of $\mathbf{W}_{bt}$ (1)

$$\max_{\mathbf{W}_{bt}} \quad R_D$$
$$\text{s.t.} \quad \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{H}_{br} \mathbf{w}_{btj} = 0, \; i \neq j, \quad \|\mathbf{W}_{bt}\|^2 \leq 1, \; \|\mathbf{W}_r \mathbf{y}_r\|^2 \leq 1.$$

- It is also a non-convex sum rate max problem. The rate tuple is again applied here.



- Define $\boldsymbol{\beta} = \{\beta_1, \cdots, \beta_{N_U}\}$, $\beta_i \geq 0$, $\sum \beta_i = 1$.
- The above problem
  $=$ Solving the above problem with constraint $R_{di} \geq \beta_i R_D$
  $+$Searching the optimal $\boldsymbol{\beta}$

Optimization of $\mathbf{W}_{bt}$ (2)

$$\max_{\mathbf{W}_{bt}} \quad R_D$$

$$\text{s.t.} \quad \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{H}_{br} \mathbf{w}_{btj} = 0, \; i \neq j, \quad \|\mathbf{W}_{bt}\|^2 \leq 1, \; \|\mathbf{W}_r \mathbf{y}_r\|^2 \leq 1,$$

$$R_{di} \geq \beta_i R_D.$$

- Use bisection method to solve the above problem. In each step, give a R_D to see if it is feasible.
- Feasibility problem can be rewritten into a SOCP problem [Luo,2006], which can be easily solved.

Optimization of $\mathbf{W}_{br}$

$$\max_{\mathbf{W}_{br}} \quad R_U$$
$$\text{s.t.} \quad \mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \ i \neq j.$$

- Since

$$R_U = \frac{1}{2} \sum\nolimits_{i=1}^{N_u} \log_2 \left(1 + \frac{P_R P_U |\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{ir}|^2}{N_0 P_R \|\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r\|^2 + N_0 \|\mathbf{w}_{bri}^T\|^2}\right)$$

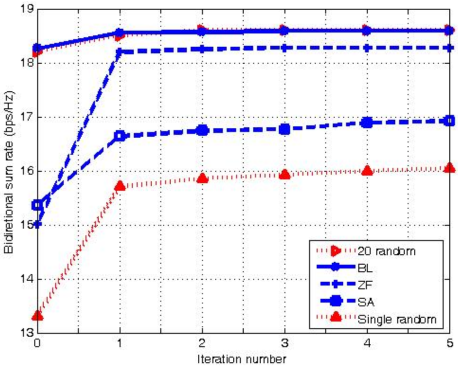
The above problem can be decoupled into N_U subproblems as

$$\max_{\mathbf{w}_{bri}} \quad \frac{P_R P_U |\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{ir}|^2}{N_0 P_R \|\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r\|^2 + N_0 \|\mathbf{w}_{bri}^T\|^2} \quad \Rightarrow \quad \max_{\mathbf{w}_{bri}} \quad \frac{\mathbf{w}_{bri}^T \mathbf{K}_S \mathbf{w}_{bri}^*}{\mathbf{w}_{bri}^T \mathbf{K}_{IN} \mathbf{w}_{bri}^*}$$
$$\text{s.t.} \quad \mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \ i \neq j. \qquad\qquad \text{s.t.} \quad \mathbf{w}_{bri}^T = \mathbf{U}_{ir}^{\perp} \mathbf{x}$$

Convergence

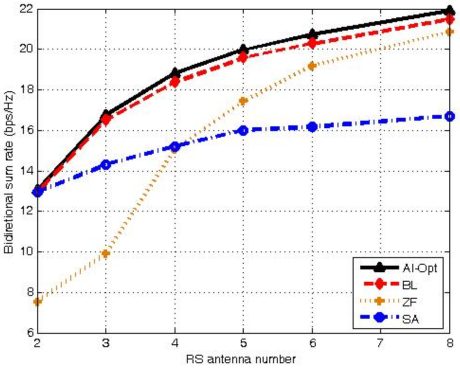
- Since we ensure the sum rate increases with iterations, it will surely converge.
- Due to the non-convexity of the original problem, the converged result depends on the initial values.
- Although a global optimal result can not be guaranteed, we increase the probability to achieve the optimal result by performing the alternating optimization with *multiple* initial values.

Simulations-Convergence



- $N_R = 4,\ N_B = N_U = 2$;
- $P_B = P_R = 2,\ P_U = 1$;
- BL: balanced scheme [Sun,2010]
- SA: signal alignment [Toh,2009, Ding,2011]
- ZF: Zero forcing

Simulations-Sum Rate



- $N_B = N_U = 2;$
- $P_B = P_R = 2, \ P_U = 1;$
- Al-opt:
  Alternating optimization

Conclusion

- We employed the alternating optimization to design the BS and RS transceiver in a two-way relay cellular system, aiming at maximizing the bidirectional sum rate under the interference-free constraints and transmit power constraints.
- In order to increase the probability to achieve the global optimal result, we used multiple initial values to perform the alternating optimization, and selected the one leading to the maximum converged sum rate. The obtained performance can be taken as a performance benchmark for the concerned system.
- The performance gap between the balance scheme and the alternating optimization is marginal, which indicates that balance scheme is a near optimal solution.

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Thank you for your attention!

Initial Values

- Initial values should satisfy all the constraints.

$$\mathbf{w}_{bri}^T \mathbf{H}_{br}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{h}_{jr} = 0, \ \mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{H}_{br} \mathbf{w}_{btj} = 0, \ i \neq j$$
$$\|\mathbf{W}_{bt}\|^2 \leq 1, \ \|\mathbf{W}_r \mathbf{y}_r\|^2 \leq 1.$$

- Rewrite that: $\mathbf{h}_{ir}^T \mathbf{W}_r \mathbf{h}_{jr} = 0 \rightarrow (\mathbf{h}_{jr}^T \otimes \mathbf{h}_{ir}^T)\mathrm{vec}(\mathbf{W}_r) = 0, \ i \neq j$
- Choose initial values of $\mathbf{W}_r$ from the general solutions $\mathrm{vec}(\mathbf{W}_r) = \mathbf{S}_\perp(\mathbf{A})\mathbf{x}$, where $\mathbf{S}_\perp$ denotes the orthogonal subspace of a matrix, $\mathbf{A} = [\mathbf{h}_{jr}^T \otimes \mathbf{h}_{ir}^T]_{i \neq j}$
- Substitute the chosen $\mathbf{W}_r$ into the other two IUI free constraints, and choose the initial values of $\mathbf{W}_{bt}$ and $\mathbf{W}_{br}$ from their general solutions respectively.
- Multiply $\mathbf{W}_r$ and $\mathbf{W}_{bt}$ with proper scalars to satisfy the power constraints.