# Front-end Feature Transforms with Context Filtering for Speaker Adaptation

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- Maximum Likelihood Context Filtering
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  - Experimental Setup
  - Results
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### Front-end Speaker Adaptation

#### Front-end Transforms:

- Linear transforms
  - Feature-space MLLR (i.e. CMLLR) [Gales'98])
  - Discriminative linear transform [Wang'03]
- Non-linear transforms ([Olsen'03, Visweswariah'04, Saon'04])

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#### FMLLR variants:

- Q-FMLLR ([Varadarajan'08])
- Full-covariance FMLLR ([Povey06, Ghoshal'08])

# Original FMLLR

### Definition (Feature-space Maximum Likelihood Linear Regression)

Given adaptation data  $x_t$ , t = 1, ..., T of a speaker, find an affine transform to maximize the likelihood of adaptation data given the current model.

$$y_t = Ax_t + b = W\xi_t$$

$$\xi_t = \left[ egin{array}{c} \mathsf{x}_t \\ 1 \end{array} 
ight]$$
: input feature extended with  $1$ 

W: extended transformation matrix  $[A \ b]$  with square matrix A of size  $d \times d$  (d is the size of  $x_t$ ) and bias term b

# Original FMLLR (cont.)

The objective function: log likelihood of the transformed data given the current model, plus the Jacobian compensation term

$$Q(W) = T \log \det(A) - \frac{1}{2} \sum_{j=1}^{G} \sum_{t=1}^{T} \gamma_{t}(j) (W \xi_{t} - \mu_{j})^{T} \Sigma_{j}^{-1} (W \xi_{t} - \mu_{j})$$

- j: index of Gaussian components
- $\mu_i, \Sigma_i$ : mean and diagonal covariance matrix
- $\gamma_t(j)$ : Gaussian occupation probabilities.

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 $\hat{x}_t = [x_{t-1} \ x_t \ x_{t+1}]$ . Find an affine transform to  $\hat{x}_t$  to maximize the likelihood

$$y_t = A\hat{x}_t + b = W\hat{\xi}_t$$

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Note: there is no direct way to compute the derivative of objective function Q(W) with respect to  $\hat{x}_t$ .



Find the Jacobian compensation term to make  $L_y + \mathcal{C} = L_x$ : simply assume y = Ax, A is square and invertible  $\implies$  compensation term  $\mathcal{C} = \frac{1}{2} \log \frac{\det(\Sigma_y)}{\det(\Sigma_x)}$ 

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#### Proof.

•

$$L_{x} = -\frac{1}{2}(x - \mu_{x})^{T} \Sigma_{x}^{-1}(x - \mu_{x}) - \frac{1}{2} \log \det(\Sigma_{x})$$

•

$$L_y = -\frac{1}{2}(y - \mu_y)^T \Sigma_y^{-1}(y - \mu_y) - \frac{1}{2} \log \det(\Sigma_y)$$

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•

$$(y - \mu_y)^T \Sigma_y^{-1} (y - \mu_y) = (x - \mu_x)^T A^T (A \Sigma_x A^T)^{-1} A (x - \mu_x)$$
  
=  $(x - \mu_x)^T \Sigma_x^{-1} (x - \mu_x)$ 

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• Set 
$$L_x = L_y + \mathcal{C}$$
,  $\mathcal{C} = \frac{1}{2} \log \frac{\det(\Sigma_y)}{\det(\Sigma_x)} = \frac{1}{2} \log \frac{\det(A\Sigma_x A^T)}{\det(\Sigma_x)} = \log \det(A)$ .

we assume the compensation term  $\mathcal{C}$  remains the same:  $\mathcal{C} = \frac{1}{2} \log \det(A \Sigma_{\hat{x}} A^T)$  (drop out  $\log \det(\Sigma_{\hat{x}})$  because it does not depend on A)

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(drop out log det( $\Sigma_{\hat{x}}$ ) because it does not depend on A) the objective function becomes:

$$Q(W) = \frac{1}{2} T \log \det(A \Sigma_{\hat{x}} A^T) - \frac{1}{2} \sum_{j=1}^{N} \sum_{t=1}^{T} \gamma_t(j) (W \hat{\xi}_t - \mu_i)^T \Sigma_j^{-1} (W \hat{\xi}_t - \mu_j)$$

The first term is a replacement of the Jacobian term when A is not a square matrix, while  $\Sigma_{\hat{x}}$  is the covariance matrix computed from  $\hat{x}_t$ , t=1,...,T.

### Compute the obj. function using stats files

### Definition (mean and variance stats)

$$K = \sum_{j=1}^{N} \sum_{t=1}^{T} \gamma_{t}(j) \Sigma_{j}^{-1} \mu_{j} \hat{\xi_{t}}^{T}, \text{ size } d \times (3d+1)$$

$$G_{i} = \sum_{j=1}^{N} \sum_{t=1}^{T} \gamma_{t}(j) \sigma_{j,l}^{-1} \hat{\xi_{t}} \hat{\xi_{t}}^{T}, \text{ size } (3d+1) \times (3d+1), i = 1, ..., d$$

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the objective function for context filtering and its gradient

$$Q(W) = \frac{1}{2} T \log \det(A\Sigma_{\hat{x}}A^{T}) + tr(W^{T}K) - \frac{1}{2} \sum_{j=1}^{n} tr(W^{T}E_{j,j}WG_{j})$$

$$\frac{\partial Q}{\partial W} = T \times [(A\Sigma_{\hat{x}}A^{T})^{-1}A\Sigma_{\hat{x}}, \quad \mathbf{0}] + K - \sum_{j=1}^{n} E_{j,j}WG_{j}$$

## Solve the optimization problem

- row-by-row iterative update algorithm in [Gales 98] cannot be applied here
  - the determinant of a square matrix equals the dot product of any given row with the corresponding row of cofactors.
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### Solve the optimization problem

- row-by-row iterative update algorithm in [Gales 98] cannot be applied here
  - the determinant of a square matrix equals the dot product of any given row with the corresponding row of cofactors.
  - It is not obvious how to extend this algorithm to non-square matrices
- use limited memory BFGS algorithm along with line search (HCL package)
  - only need functions to evaluate the objective function and its gradient
  - gradient magnitude/maximum number of iterations are set to stop the opt. module

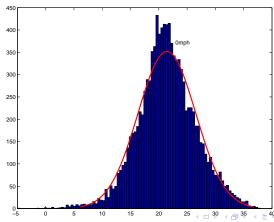
# Training data and Models

- Majority of the training data was collected in stationary cars
- Total 800K training utterances/800 hours
- Word-internal with pentaphone context, with 830 context-dependent states and 10K Gaussians
- With LDA 40-dim features, built a ML model, a discriminative trained BMMI (boosted maximum mutual information) model, and a FMMI (feature-space maximum mutual information) model.

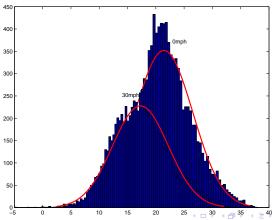
#### Test data

- Recorded in cars at three different speeds: 0mph (idling), 30mph and 60mph.
- Four tasks are selected in the test set: addresses, digits, commands and radio control
- Total about 26K utterances and 130K words

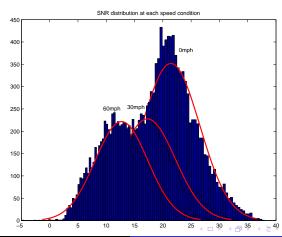
### SNR distribution



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### Experiments

- Unsupervised speaker adaptation on the corresponding ML/BMMI/FMMI models
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- Unsupervised speaker adaptation on the corresponding ML/BMMI/FMMI models
- MLCF-n: maximum likelihood context filtering with context size n
- MLCF transform initialization: zero matrices for all the frames/identity matrix for the center
- MLCF-n-init: uses FMLLR as the starting point for the center frame

WER/SER	0mph	30mph	60mph
baseline	0.77/3.34	1.28/5.15	2.65/8.94

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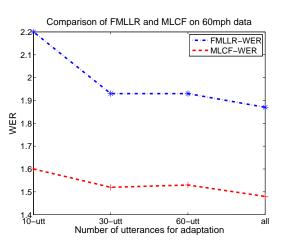
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MLCF-2	0.55/2.41	0.93/3.84	1.44/5.49

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MLCF-2	0.55/2.41	0.93/3.84	1.44/5.49
MLCF-1	0.54/2.31	0.95/3.91	1.48/5.54
MLCF-1-init	0.54/2.32	0.96/3.89	1.50/5.58

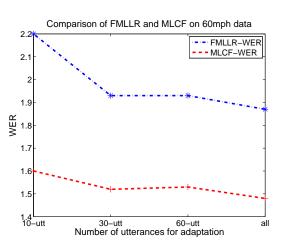
Table: Comparison of FMLLR and MLCF adapted on the ML model.

- Compared to FMLLR, on noisy 60mph data, 23%/13% relative gain on WER/SER over FMLLR, tiny gains on 0mph/30mph
- Starting with FMLLR for the center frame does not provide any advantage over the identity matrix

### Effect of adaptation data

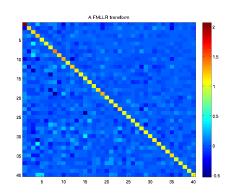


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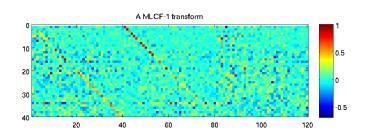


- in 10-utt case, MLCF-1 gains even more over FMLLR — 30% relative
- for FMLLR there is 15% degradation from all-utterance to 10-utterance, while for MLCF-1, only 7% relative degradation.

### Visual comparison of FMLLR and MLCF



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WER/SER	0mph	30mph	60mph
baseline	0.63/2.76	0.96/3.82	2.02/6.95
FMLLR	0.46/1.98	0.75/3.06	1.47/5.24
MLCF-1	0.45/1.91	0.74/3.05	1.33/4.68
MLCF-1-init	0.43/1.86	0.74/3.04	1.33/4.77

Table: Comparison of FMLLR and MLCF adapted on the BMMI model.

- $\bullet$  9%/11% relative improvement of WER/SER over FMLLR on the noisy 60mph data
- starting with FMLLR transform for the central frame does not provide any advantage over the identity transform.

WER/SER	0mph	30mph	60mph
baseline	0.45/1.90	0.76/3.23	1.30/5.05
FMLLR	0.33/1.40	0.60/2.52	1.00/4.06
MLCF-1	0.32/1.31	0.61/2.56	0.96/3.84
MLCF-1-init	0.32/1.34	0.59/2.47	0.93/3.75

Table: Comparison of FMLLR and MLCF adapted on the FMMI model.

This time MLCF-1-init is better than MLCF-1, and gains 7%/9% relative on WER/SER over FMLLR.

### Summary

- MLCF: extend the full-rank square matrix of FMLLR to a non-square matrix that uses neighboring feature vectors to estimate the adapted central feature vector
- MLCF is shown outperform FMLLR on noisy 60mph data:
   23% on WER over FMLLR with adapted ML model, and
   7%/9% on the FMMI/BMMI models.

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#### Future work includes

- use disc. ojective function or smoothing of disc. and ML objective functions
- check the interaction of context filtering with other front-end noise robustness techniques (e.g. Spectral Substraction, Dynamic Noise Adaptation)

