P and T Wave Delineation and Waveform Estimation in ECG Signals Using a Block Gibbs Sampler

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Outline

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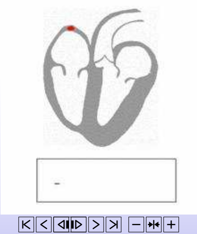
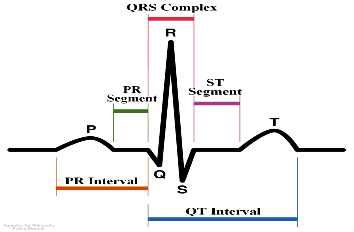
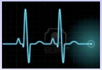
Electrocardiogram (ECG)

- A recording of the electrical activity of the heart over time
- 3 distinct waves are produced during cardiac cycle
  - P wave caused by atrial depolarization
  - QRS complex caused by ventricular depolarization
  - T wave results from ventricular repolarization and relax
- Wave shapes and interval durations indicate clinically useful information







ECG delineation

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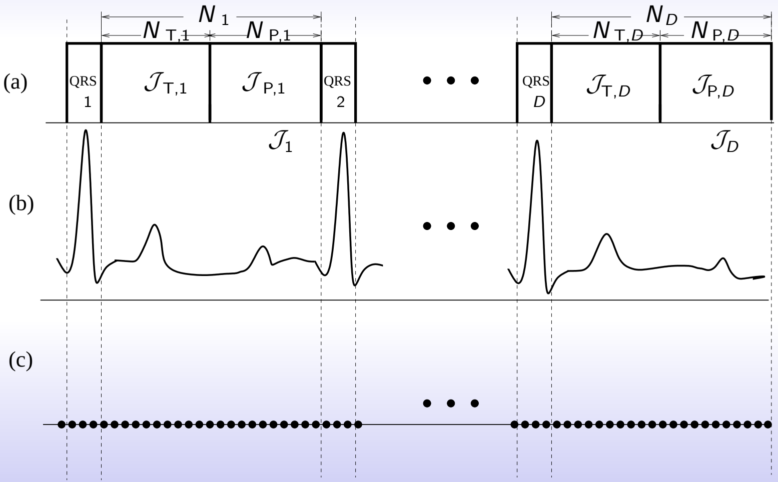
- P and T wave delineation—a challenging problem

ECG delineation

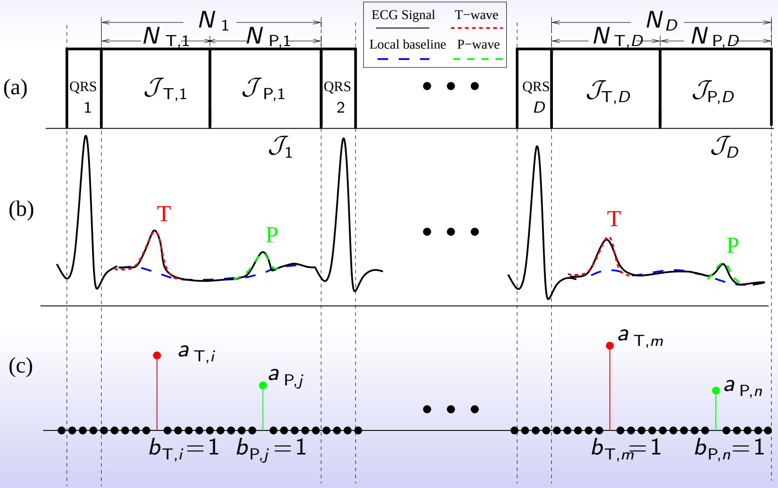
- Delineation: determination of peaks and boundaries of the waves
- P and T wave delineation—a challenging problem
- Existing methods
  - Filtering: adaptive filtering, nested median filtering, low pass differentiation (LPD)
  - Basis expansions: Fourier transform, discrete cosine transform, wavelet transform (WT)
  - Classification and pattern recognition: fuzzy theory, hidden Markov models, artificial neural networks
  - Bayesian inference: off line and sequential approaches (Kalman filter)

Outline

Signal model for the non-QRS intervals



Signal model for the non-QRS intervals



Signal model for the non-QRS intervals

non-QRS signal components within a D-beat window

$$x_k = \sum_{l=-L/2}^{L/2} h_{\mathrm{T},l} u_{\mathrm{T},k-l} + \sum_{l=-L/2}^{L/2} h_{\mathrm{P},l} u_{\mathrm{P},k-l} + c_k + w_k, \quad k \in \mathcal{J}^*, \tag{1}$$

- $u_{\mathrm{T},k} = b_{\mathrm{T},k} a_{\mathrm{T},k}$: unknown "impulse" sequence indicating T wave locations and amplitudes,
- $u_{\mathrm{P},k} = b_{\mathrm{P},k} a_{\mathrm{P},k}$: unknown "impulse" sequence indicating P wave locations and amplitudes,
- $\mathbf{h}_\mathrm{T} = (h_{\mathrm{T},-L/2} \cdots h_{\mathrm{T},L/2})^T$: unknown T waveform,
- $\mathbf{h}_\mathrm{P} = (h_{\mathrm{P},-L/2} \cdots h_{\mathrm{P},L/2})^T$: unknown P waveform,
- c_k: baseline sequence, w_k: white Gaussian noise

Signal model for the non-QRS intervals

- Representation of the P and T waveforms by a Hermite basis expansion

$$\mathbf{h}_{\mathrm{T}} = \mathbf{H}\boldsymbol{\alpha}_{\mathrm{T}}\,, \quad \mathbf{h}_{\mathrm{P}} = \mathbf{H}\boldsymbol{\alpha}_{\mathrm{P}}\,,$$

  - $\mathbf{H}$ is a $(L+1) \times G$ matrix whose columns are the first G Hermite functions with $G \leq (L+1)$
  - $\boldsymbol{\alpha}_{\mathrm{T}}$ and $\boldsymbol{\alpha}_{\mathrm{P}}$ are unknown coefficient vectors of length G

- Modeling of the local baseline within the n-th non-QRS interval by a $4th$-degree polynomial

$$\mathbf{c}_n = \mathbf{M}_n \boldsymbol{\gamma}_n\,,$$

  - $\mathbf{M}_n$ is the known $N_n \times 5$ Vandermonde matrix
  - $\boldsymbol{\gamma}_n = (\gamma_{n,1} \cdots \gamma_{n,5})^T$ is the unknown coefficient vector

Signal model for the non-QRS intervals

vector representation of the non-QRS signal in (1)

$$\mathbf{x} = \mathbf{F}_{\mathrm{T}}\mathbf{B}_{\mathrm{T}}\mathbf{a}_{\mathrm{T}} + \mathbf{F}_{\mathrm{P}}\mathbf{B}_{\mathrm{P}}\mathbf{a}_{\mathrm{P}} + \mathbf{M}\boldsymbol{\gamma} + \mathbf{w}\,, \tag{2}$$

- $\mathbf{b}_{\mathrm{T}}$, $\mathbf{b}_{\mathrm{P}}$, $\mathbf{a}_{\mathrm{T}}$, and $\mathbf{a}_{\mathrm{P}}$ denote the $M \times 1$ vectors corresponding to $b_{\mathrm{T},k}$, $b_{\mathrm{P},k}$, $a_{\mathrm{T},k}$, and $a_{\mathrm{P},k}$, respectively.
- $\mathbf{B}_{\mathrm{T}} \triangleq \mathrm{diag}(\mathbf{b}_{\mathrm{T}})$ and $\mathbf{B}_{\mathrm{P}} \triangleq \mathrm{diag}(\mathbf{b}_{\mathrm{P}})$,
- $\mathbf{F}_{\mathrm{T}}$ and $\mathbf{F}_{\mathrm{P}}$ are the $K \times M$ Toeplitz matrices with first row $\left(\mathbf{h}_1^T\boldsymbol{\alpha}_{\mathrm{T}} \;\; \mathbf{0}_{M-1}^T\right)$ and $\left(\mathbf{h}_1^T\boldsymbol{\alpha}_{\mathrm{P}} \;\; \mathbf{0}_{M-1}^T\right)$, respectively.
- $\mathbf{M}$, and γ are obtained by concatenating the $\mathbf{M}_n$ and γ_n, for $n = 1, \ldots, D$.

Outline

Model parameters

Bayesian estimation relies on the posterior distribution

$$p(\boldsymbol{\theta}|\mathbf{x}) \propto p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})$$

$\boldsymbol{\theta} = (\boldsymbol{\theta}_{\mathrm{T}}^T\, \boldsymbol{\theta}_{\mathrm{P}}^T\, \boldsymbol{\theta}_{\mathrm{cw}}^T)^T$ are the unknown parameters resulting from (2)

- $\boldsymbol{\theta}_{\mathrm{T}} \triangleq (\mathbf{b}_{\mathrm{T}}^T\, \mathbf{a}_{\mathrm{T}}^T\, \boldsymbol{\alpha}_{\mathrm{T}}^T)^T$ and $\boldsymbol{\theta}_{\mathrm{P}} \triangleq (\mathbf{b}_{\mathrm{P}}^T\, \mathbf{a}_{\mathrm{P}}^T\, \boldsymbol{\alpha}_{\mathrm{P}}^T)^T$ are T and P wave related parameter vectors,
- $\boldsymbol{\theta}_{\mathrm{cw}} \triangleq (\boldsymbol{\gamma}^T\, \sigma_w^2)^T$ are baseline and noise parameters.

Likelihood function

$$p(\mathbf{x}|\boldsymbol{\theta}) \propto \frac{1}{\sigma_w^K} \exp\left(-\frac{1}{2\sigma_w^2}\|\mathbf{x} - \mathbf{F}_{\mathrm{T}}\mathbf{B}_{\mathrm{T}}\mathbf{a}_{\mathrm{T}} - \mathbf{F}_{\mathrm{P}}\mathbf{B}_{\mathrm{P}}\mathbf{a}_{\mathrm{P}} - \mathbf{M}\boldsymbol{\gamma}\|^2\right),$$

where $\|\cdot\|$ is the ℓ_2 norm, i.e., $\|\mathbf{x}\|^2 = \mathbf{x}^T\mathbf{x}$.

Location prior

T wave indicator prior: block constraint

$$p(\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}) = \begin{cases} p_0 & \text{if } \|\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}\| = 0 \\ p_1 & \text{if } \|\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}\| = 1 \\ 0 & \text{otherwise,} \end{cases}$$

Assuming independence between consecutive non-QRS intervals, the prior of $\mathbf{b}_{\mathrm{T}}$ is given by

$$p(\mathbf{b}_{\mathrm{T}}) = \prod_{n=1}^{D} p(\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}).$$

Amplitude and waveform priors

T wave amplitude prior

$$p(a_{\mathrm{T},k}|b_{\mathrm{T},k}=1) = \mathcal{N}(a_{\mathrm{T},k}; 0, \sigma_a^2)$$

- $a_{\mathrm{T},k}$ are only defined at time instants k where $b_{\mathrm{T},k}=1$,
- $u_{\mathrm{T},k}=b_{\mathrm{T},k}a_{\mathrm{T},k}$ is a Bernoulli-Gaussian sequence with block constraints.

T waveform coefficients prior

$$p(\boldsymbol{\alpha}_{\mathrm{T}}) = \mathcal{N}(\boldsymbol{\alpha}_{\mathrm{T}}; \mathbf{0}, \sigma_\alpha^2 \mathbf{I}_{L+1})$$

The priors of the P wave parameters $\mathbf{b}_{\mathrm{P}}$, $\mathbf{a}_{\mathrm{P}}$ and $\boldsymbol{\alpha}_{\mathrm{P}}$ are defined in a fully analogous way!

Baseline and noise variance priors

Baseline coefficient prior

$$p(\gamma) = \mathcal{N}(\gamma; \mathbf{0}, \sigma_\gamma^2 \mathbf{I}_{5D})$$

Noise variance prior

$$\sigma_w^2 = \mathcal{IG}\left(\sigma_w^2; \xi, \eta\right)$$

Conjugate priors for simplicity

Posterior distribution

$$p(\boldsymbol{\theta}|\mathbf{x}) \propto p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}) = p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta}_{\mathrm{T}})p(\boldsymbol{\theta}_{\mathrm{P}})p(\boldsymbol{\theta}_{\mathrm{cw}})$$

Complex distribution

Outline

Block Gibbs sampler

- In a D-beat processing window, for each non-QRS interval:
  - Sample the T indicator block $\mathbf{b}_{\mathcal{J}_{\mathrm{T},n}}$
  - For the k where $b_{\mathrm{T},k} = 1$, sample the T amplitudes $a_{\mathrm{T},k}$
  - Sample the P indicator block $\mathbf{b}_{\mathcal{J}_{\mathrm{P},n}}$
  - For the k where $b_{\mathrm{P},k} = 1$, sample the P amplitudes $a_{\mathrm{P},k}$
- Sample P and T waveform coefficients $\boldsymbol{\alpha}_{\mathrm{T}}$ and $\boldsymbol{\alpha}_{\mathrm{P}}$
- Sample baseline coefficients $\boldsymbol{\gamma}$
- Sample noise variance σ_w^2

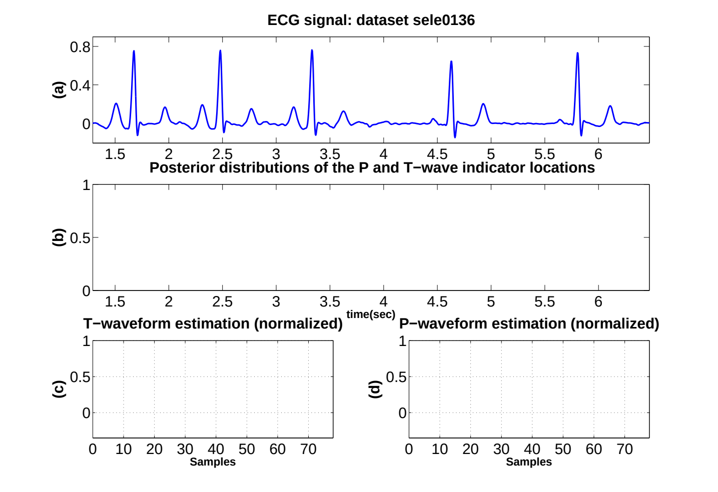
Outline

Simulation parameters

- Preprocessing: QRS complexes detection using the algorithm of **Pan et al.** (*IEEE Trans. Biomed. Eng.*, 1985),
- Processing window length: $D = 10$,
- For each estimation, the 40 first iterations are disregarded (burn-in period) and 60 iterations are used to compute the estimates.
- Real ECG datasets from the QT database.
- Computation time: 8 seconds to run 100 iterations on a 10-beat ECG block (Matlab implementation).

C. Lin et al., P- and T-wave delineation in ECG signals using a Bayesian approach and a partially collapsed Gibbs sampler, *IEEE Trans. Biomed. Eng.*, 2010

Typical examples



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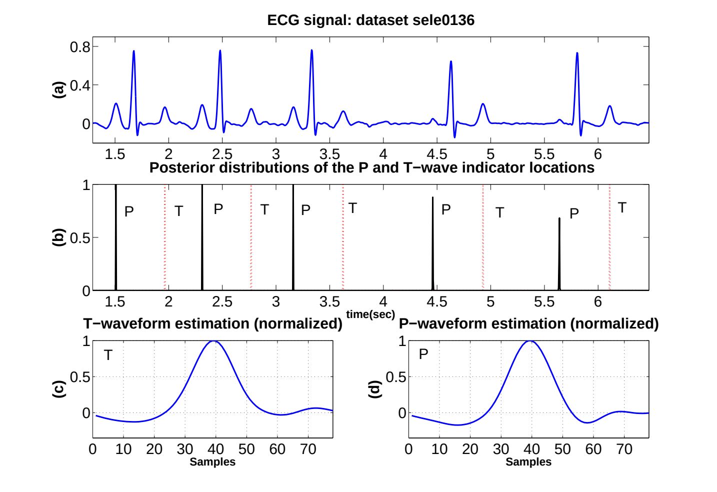

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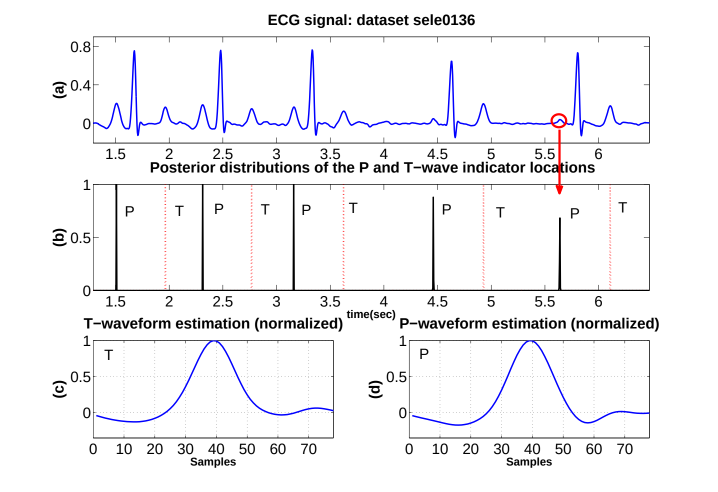

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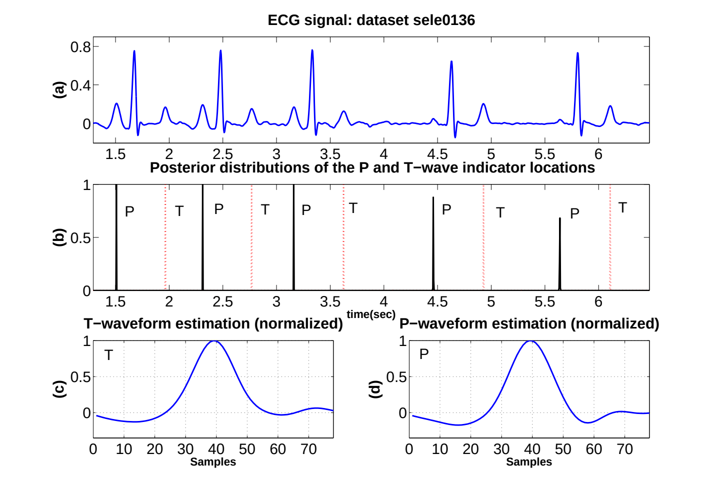

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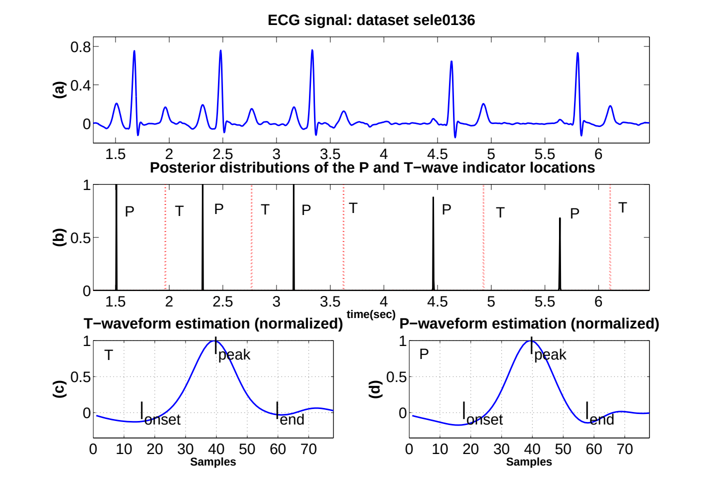

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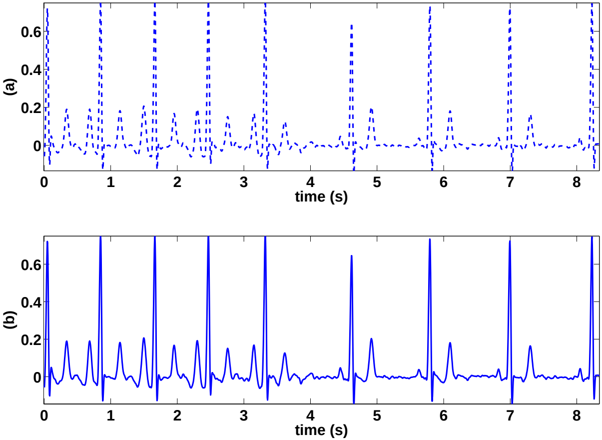

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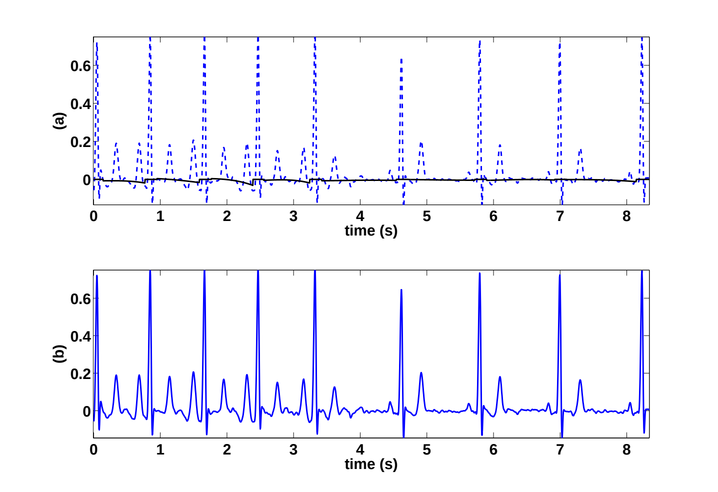

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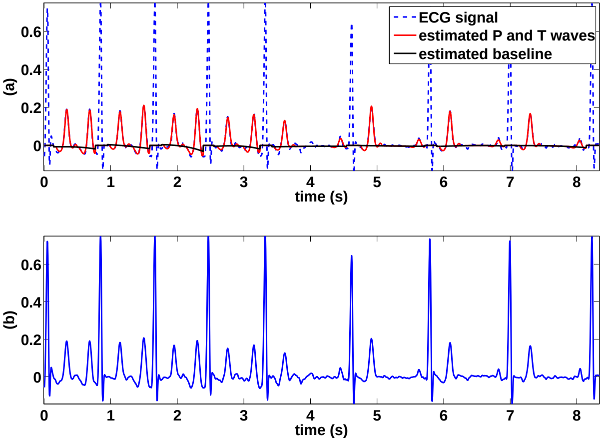

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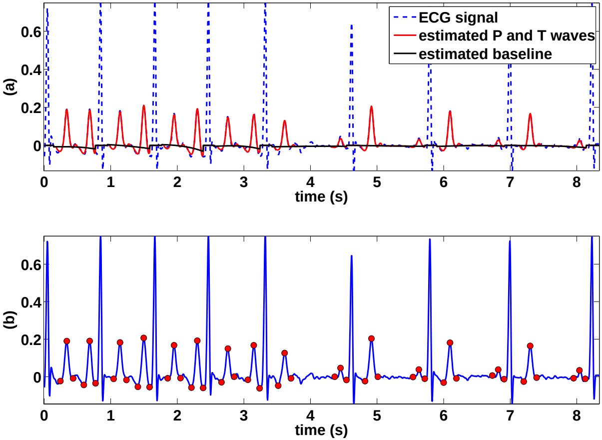
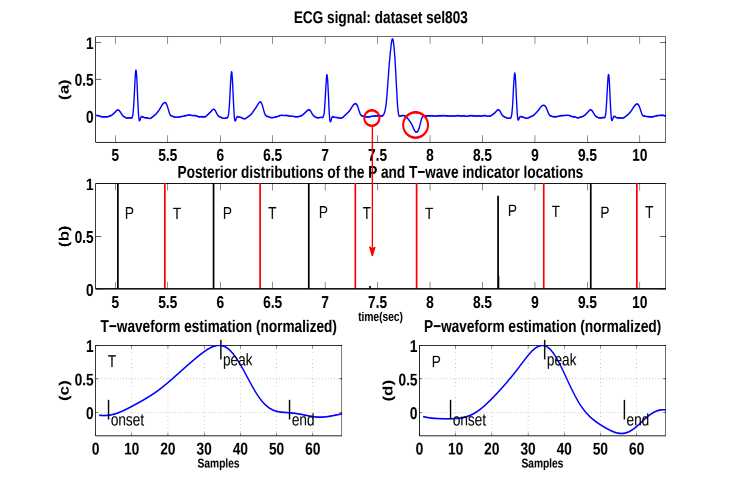


Premature ventricular contraction ECG



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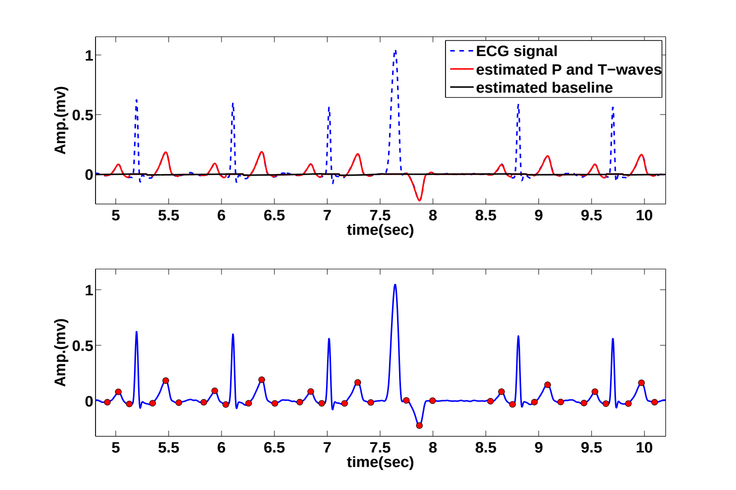
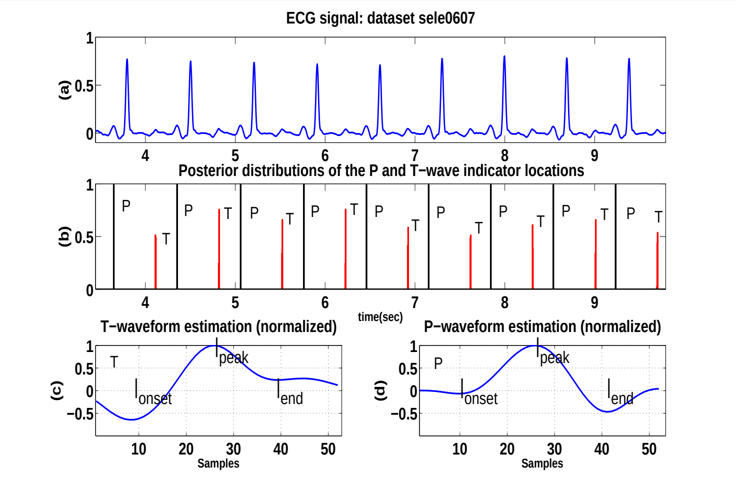


Biphasic T wave ECG



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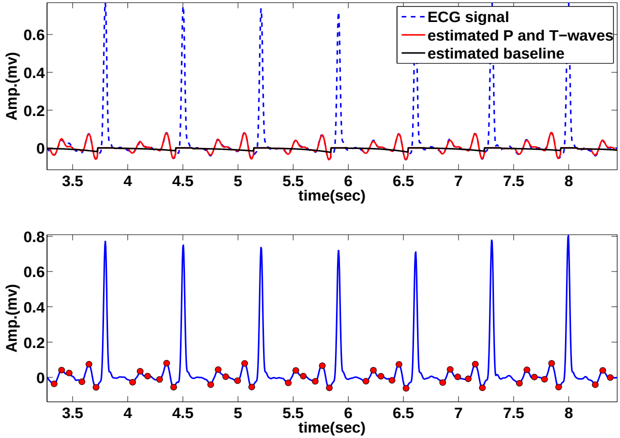
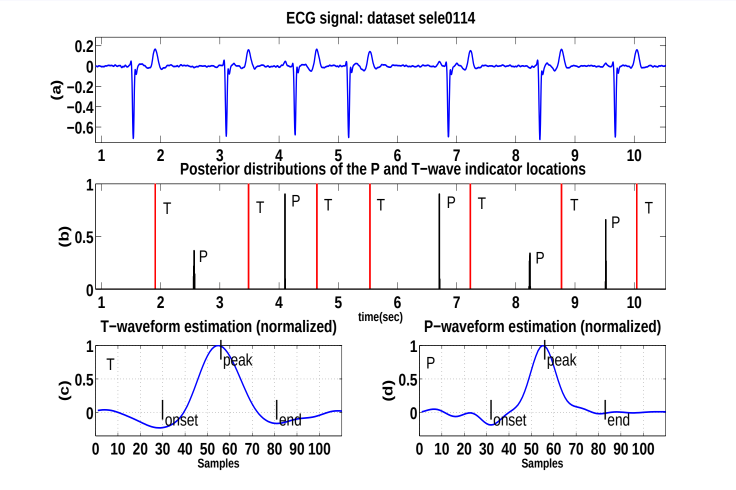


Absence of P wave



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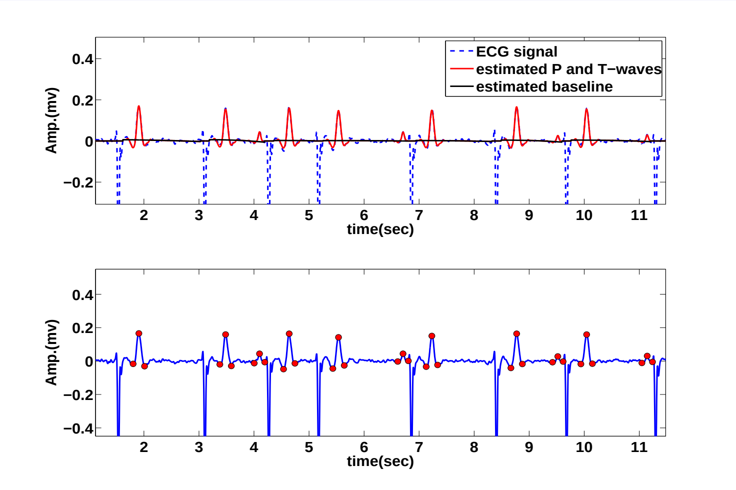

Evaluation on QTDB

Table: Delineation and detection performance for QTDB

Parameter		Proposed alg.	LPD	WT
b_P:	Se (%)	99.60	97.70	98.87
Onset-P:	$\mu \pm \sigma$ (ms)	1.7 ± 10.8	14.0 ± 13.3	2.0 ± 14.8
Peak-P:	$\mu \pm \sigma$ (ms)	2.7 ± 8.1	4.8 ± 10.6	3.6 ± 13.2
End-P:	$\mu \pm \sigma$ (ms)	2.5 ± 11.2	-0.1 ± 12.3	1.9 ± 12.8
b_T:	Se(%)	100	97.74	99.77
Onset-T:	$\mu \pm \sigma$ (ms)	5.7 ± 16.5	N/A	N/A
Peak-T:	$\mu \pm \sigma$ (ms)	0.7 ± 9.6	-7.2 ± 14.3	0.2 ± 13.9
End-T:	$\mu \pm \sigma$ (ms)	2.7 ± 13.5	13.5 ± 27.0	-1.6 ± 18.1

Outline

Conclusion and future works

Conclusion

- A Bayesian model for the non-QRS intervals of ECG signals,
- A block Gibbs sampler for joint delineation and waveform estimation of P and T waves,
- Evaluation on the QTDB is promising.

Prospects

- Exploitation of the amplitude estimation, ex., TWA detection,
- Exploitation of the waveform estimation, ex., arrhythmia detection,
- Beat-to-beat / sequential delineation.

Thank you for your attention!

Matlab demo available at
`http://www.enseeiht.fr/~lin`

Outline

Time-shift and scale ambiguities

Issue: No unique solution for a convolution model

- Scale ambiguity: $\mathbf{h} \star \mathbf{u} = (a\mathbf{h}) \star (\mathbf{u}/a), \ \forall a \neq 0$,
- Time-shift ambiguity: $\mathbf{h} \star \mathbf{u} = (d_\tau \star \mathbf{h}) \star (d_{-\tau} \star \mathbf{u}), \ \forall \tau \in \mathbb{Z}$.

Solution: Hybrid Gibbs sampling

- Metropolis-Hastings within Gibbs after sampling waveform coefficients,
- Deterministic shifts after sampling waveform coefficients:
  - Time-shifts to have $h_0' = \max |\mathbf{h}|$,
  - Scale-shifts to have $h_0' = 1$,

C. Labat et al., Sparse blind deconvolution accounting for time-shift ambiguity, *ICASSP*, 2006

References

Similar applications on physiological signal processing:

- on OCT signals: **G. Kail et al.**, A blind Monte Carlo detection-estimation method for optical coherence tomography, *ICASSP*, 2009
- on ECG signals: **C. Lin et al.**, P- and T-wave delineation in ECG signals using a Bayesian approach and a partially collapsed Gibbs sampler, *IEEE Trans. Biomed. Eng.*, 2010
- on EMG signals: **D. Ge et al.**, Spike sorting by stochastic simulation, *IEEE Trans. Neural. Syst. Rehabil. Eng.*, 2011

Other P and T wave delineation methods:

- LPD: **P. Laguna et al.**, Automatic detection of wave boundaries in multilead ECG signals: Validation with the CSE database, *Comput. Biomed. Res.*, 1994
- WT: **J. P. Martínez et al.**, A wavelet-based ECG delineator: Evaluation on standard databases, *IEEE Trans. Biomed. Eng.*, 2004
- KF: **O. Sayadi et al.**, A model-based Bayesian framework for ECG beat segmentation, *J. Physiol. Meas.*, 2009